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PROBLEMS

IN

CURVED AND INDIRECT FIRE

BY

JAMES M. INGALLS

Major, First Artillery

Fort Monroe, Va.

ARTILLERY SCHOOL PRESS,
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PROBLEMS IN CURVED AND INDIRECT FIRE.

The importance of indirect fire in siege operations has increased greatly during the last few years by the introduction of rifled guns, and especially of rifled howitzers and mortars; and also by the simultaneous improvement of ballistic formulas and tables whereby the art of gunnery has been reduced to an exact science. It is no longer considered practicable or desirable to construct breaching batteries as formerly on the glacis in order to batter down the scarp by direct fire, but the breach must now be effected by indirect fire from a distance of several hundred yards, as at the siege of Strassburg, during the Franco-German war of 1870-71, where the breach of the main rampart was made by indirect fire, the masonry not being visible from the demolition batteries located in the second parallel.

Curved fire is also considered by our best military writers as equaly important in operations against troops protected by the earthen parapets (breast works) inaugurated in our civil war. These were usualy thrown up in a single night and rendered the defenders comparatively safe against the direct fire of the enemy, whether of muskets or of field guns.

The admirable article by Major Tiedeman, on the necessity of employing curved fire against troops protected by field intrenchments, has been rendered accessible to our artillery officers through Lieutenant Blakely's translation printed in Vol. IX, of the *Journal of the U. S. Artillery*, and makes it unnecessary for me to deduce any argument to show the great importance of curved fire, or to cite examples of its successful use in battles and sieges.

Assuming then that curved fire will be a necessity in future field and siege operations it becomes all important to deduce formulas and prepare tables which shall enable the artillery officer to lay his guns quickly and with certainty under all circumstances that may arise. The ordinary range tables are here of but little use since the muzzle velocity is generally one of the elements to be determined, and in the great majority of cases will be less than 800 f. s. to secure the required angle of fall. The selection of the proper guns for curved firing against troops protected by earth works, or, if the ordinary field gun be employed, the method of reducing the charge so as to secure the desired muzzle velocity, are problems for the consideration of the Ordnance Department. We shall deduce formulas and compute a concise table by means of which the necessary muzzle velocity, range, angle of departure and time of flight to secure the desired results, can be easily and accurately computed.

Problems in curved fire have usually been solved by means of the tables deduced by Siacci and Lardillon from those computed in 1842 by the Prussian General Otto, after Euler's Method.* But these solutions are not so convenient as those which we shall deduce from Siacci's equations for direct fire. These primary equations, which are now well known to all Artillerists, are the following: †

$$x = C\left\{S(u) - S(V)\right\} \tag{1}$$

$$\frac{y}{x} = \tan\varphi - \frac{C}{2\cos^2\varphi}\left\{\frac{A(u) - A(V)}{S(u) - S(V)} - I(V)\right\} \tag{2}$$

$$\tan\theta = \tan\varphi - \frac{C}{2\cos^2\varphi}\left\{I(u) - I(V)\right\} \tag{3}$$

$$t = C\sec\varphi\left\{T(u) - T(V)\right\} \tag{4}$$

$$u = v\cos\theta\sec\varphi \tag{5}$$

$$C = f\frac{\delta_1 w}{\delta c \beta d^2} \tag{6}$$

In these equations

V is the velocity of the projectile as it emerges from the gun, called the muzzle velocity.

φ is the angle of projection, that is the angle which the tangent to the trajectory at the muzzle makes with the axis of x.

x, y are the rectangular coordinates of any point of the trajectory, the origin being at the muzzle of the gun and the axis of x horizontal.

v is the velocity of the projectile at the point x, y.

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* For a full explanation of this method, with a modified form of table, see Ingalls' *Handbook of Problems in Direct Fire*, 1890, pp. 285-296.

† For a demonstration of these formulas, see *Handbook*, Appendix 1.

θ is the angle which the tangent to the trajectory at the point x, y makes with the axis of x.

t is the time of flight of the projectile from the origin to the point x, y.

u is called by Siacci the *pseudo velocity* and is employed in nearly all ballistic equations in place of the true velocity v. It is this use of the *pseudo* velocity instead of the *actual* velocity that constitutes the real essence of Siacci's method for direct fire.

C is called the *ballistic coefficient* and depends upon the weight, diameter and shape of head of projectile, its steadiness in flight and the density of the air it encounters. When properly determined it measures the ability of a projectile to overcome the resistance of the air: or, as we may say, it measures the *ballistic efficiency* of a projectile. Of the factors which make up the ballistic coefficient.

f is called the *altitude factor*, and takes into account the diminution of resistance encountered by the projectile as it rises above the earth's surface. It is greater than unity and increases with the mean altitude of the trajectory which is taken at two-thirds the maximum altitude. Its value is given by the formula*

$$\log(\log f) = \log y_0 - 4.98235 \tag{7}$$

in which y_0 is the maximum ordinate, or the height of the summit of the trajectory, in feet. For low trajectories this factor may be taken as unity.

$\frac{\delta_1}{\delta}$ is a factor depending upon the varying density of the air near the earth's surface. The value of this factor for any observed temperature (Fahrenheit) and barometric pressure (inches of mercury) may be taken from Table III.

c is called the *coefficient of reduction*, and is unity for guns and projectiles similar to those used in making the experiments upon which the ballistic tables are based. If the qualities of the gun and projectile are such that the latter meets with a greater resistance than the experimental projectile of the same diameter the effect is the same as if the area of cross section were increased, and therefore in this case c must be greater than unity. On the other hand if the resistance a projectile encounters be less than the experimental resistance for the same cross section than for this projectile c will be less than unity. For modern guns firing projectiles having heads struck with radii of two calibers c will be taken as unity.

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* See *Handbook*, pages 87-88 and 307.

β is Siacci's *integrating factor*, and its presence in the expression for the ballistic coefficient corrects the effect of certain erroneous assumptions which were necessarily made in deducing the primary equations, when the curvature of the trajectory is considerable. The values of these factors (which are functions of the angle of projection and range) are given in Table 2. They are taken from Siacci's paper *Nuova tavola della funzione* β.*

$S(V)$, $S(u)$, $A(V)$, $A(u)$, $I(V)$, $I(u)$, $T(V)$, $T(u)$ are certain functions of the muzzle and pseudo velocities, called, respectively, space function, altitude function, inclination function and time function. For their analytical expressions the reader is referred to any standard work on exterior ballistics.†

If we consider the entire horizontal trajectory, that is, that part of the complete trajectory lying above the level of the gun, and designate the horizontal range by X, the corresponding time of flight by T, and the angle of fall (considered positive) by ω, the primary equations become ‡

$$X = C\left\{ S(u) - S(V) \right\} \tag{8}$$

$$\sin 2\varphi = C\left\{ \frac{A(u) - A(V)}{S(u) - S(V)} - I(V) \right\} \tag{9}$$

$$\tan \omega = \frac{C}{2\cos^2\varphi}\left\{ I(u) - \frac{A(u) - A(V)}{S(u) - S(V)} \right\} \tag{10}$$

$$T = C\sec\varphi \left\{ T(u) - T(V) \right\} \tag{11}$$

$$u = v\cos\omega\sec\varphi \tag{12}$$

With the help of Siacci's integrating factor these equations for the horizontal trajectory are practically correct for all elevations up to 45°. They can be very much simplified by the following substitutions due to Braccialini :§

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* *Rivista d'Artiglieria e Genio* for February, 1896.

† See also *Ingalls' Handbook*, Appendix 1.

‡ For convenience of printing, the letters v and u will refer to the far end of any arc under consideration. The conditions of the problem will always indicate what arc is meant.

§ *Giornale d'Artiglieria e Genio*, part 2, 1883.

$$z = S(u) - S(V)$$
$$A = \frac{A(u) - A(V)}{S(u) - S(V)} - I(V)$$
$$B = I(u) - \frac{A(u) - A(V)}{S(u) - S(V)}$$
$$A' = A + B = I(u) - I(V)$$
$$T' = T(u) - T(V)$$

Making these substitutions in equations (8) to (11) we have, for any given muzzle velocity V for which these functions have been computed,

$$X = C z \tag{13}$$
$$\sin 2\varphi = A\,C \tag{14}$$
$$\tan\omega = \frac{B\,C}{2\cos^2\varphi} \tag{15}$$
$$T = C\,T' \sec\varphi \tag{16}$$

Equation (15) may also be written for flat trajectories, that is, when the ratio

$$\frac{\cos\varphi}{\cos\omega}$$

is approximately unity,

$$\sin 2\omega = B\,C \tag{17}$$

When this ratio cannot be regarded as unity we may have, by combining equations (14) and (15) and making

$$B' = \frac{B}{A}$$

the following expression for computing ω which is appliable in all cases:

$$\tan\omega = B' \tan\varphi \tag{18}$$

The values of the space. altitude, inclination and time functions, and the secondary functions derived from them, depend for any given muzzle velocity and range, upon the law of resistance.

It is generally admitted by writers on exterior ballistics that the quadratic law of resistance holds for all velocities below 800 f. s. (240 m. s.);* and this includes nearly all firing with mortars mountain howitzers and also the indirect fire of field and siege guns and field howitzers. For this law the ballistic functions $S(u)$, $A(u)$, $I(u)$ and $T(u)$ take the following forms:†

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* Siacci, *Balistique*, French edition, 1892, page 84. La Llave, *Balistica Abreviada*, 1893, page 77. Zaboudski, *Exterior Ballistics* (in Russian), 1895, page 55. Bashforth, *The Bashforth Chronograph*, 1890, page 138.

† *Handbook*, pages 271-274.

$$S(u) = -\frac{\log_e u}{A_1} + Q_s$$

$$I(u) = \frac{g}{A_1 u^2} + Q_i$$

$$A(u) = \frac{g}{2(A_1 u)^2} - \frac{Q_i}{A_1} \log_e u + Q_a$$

$$T(u) = \frac{1}{A_1 u} + Q_t$$

and similar expressions for $S(V)$, $A(V)$, $I(V)$ and $T(V)$. Q_s, Q_i, Q_a and Q_t are constants of integration and A_1 is a constant determined by experiment. The value of A_1 which we have adopted is that deduced by Mayevski from a discussion of the Krupp firings at Meppen.* Its logarithm for English units is

$$\log A_1 = 5.6698914 - 10$$

Substituting these values in the expressions for z, A, A', B and T' the constants of integration disappear by subtraction, and we have by easy reductions

$$z = \frac{1}{A_1} \log_e \left(\frac{V}{u}\right) \tag{19}$$

$$A = \frac{g}{A_1}\left\{\frac{1}{2A_1 z}\left(\frac{1}{u^2} - \frac{1}{V^2}\right) - \frac{1}{V^2}\right\} \tag{20}$$

$$B = \frac{g}{A_1}\left\{\frac{1}{u^2} - \frac{1}{2A_1 z}\left(\frac{1}{u^2} - \frac{1}{V^2}\right)\right\} \tag{21}$$

$$A' = \frac{g}{A_1}\left(\frac{1}{u^2} - \frac{1}{V^2}\right) \tag{22}$$

$$T' = \frac{1}{A_1}\left(\frac{1}{u} - \frac{1}{V}\right) \tag{23}$$

These equations may be still further simplified by expressing them exponentially as follows: Making

$$2A_1 z = n$$

equation (19) gives

$$\frac{1}{u} = \frac{e^{\frac{1}{2}n}}{V} \therefore u = \frac{V}{e^{\frac{1}{2}n}} \tag{24}$$

and

$$\frac{1}{u^2} = \frac{e^n}{V^2}$$

Substituting these in equations (20), (21), (22) and (23) gives

$$A = \frac{g}{A_1 V^2}\left(\frac{e^n - 1}{n} - 1\right) \tag{25}$$

---

* *Revue d'Artillerie*, April, 1883. Ingalls' *Exterior Ballistics*, page 28. Zaboudski's *Exterior Ballistics* (in Russian), page 55.

$$B = \frac{g}{A_1 V^2}\left(e^{n} - \frac{e^{n} - 1}{n}\right) \qquad (26)$$

$$A' = \frac{g}{A_1 V^2}\left(e^{n} - 1\right) \qquad (27)$$

$$T' = \frac{1}{A_1 V}\left(e^{\frac{1}{2}n} - 1\right) \qquad (28)$$

Dividing equation (26) by equation (25) gives

$$B' = \frac{1 - (1 - n)\, e^{n}}{e^{n} - n - 1} \qquad (29)$$

an equation independent of the muzzle velocity V.

In addition to the secondary functions due to Braccilini we have introduced the following into Table 1:

$$A'' = \frac{B z}{A'} = z\left(\frac{1 - (1 - n)\, e^{n}}{n\,(e^{n} - 1)}\right) \qquad (30)$$

$$B'' = \sqrt{200\, g\, z}\left(\frac{e^{-n} + n - 1}{n^2}\right)^{\frac{1}{2}} \qquad (31)$$

$$C' = \frac{A}{z} = \frac{2g}{V^2}\left(\frac{e^{n} - 1 - n}{n^2}\right) \qquad (32)$$

$$C'' = \frac{A}{2\,T'} = \frac{g}{2V}\left(\frac{e^{n} - 1 - n}{n\,(e^{\frac{1}{2}n} - 1)}\right) \qquad (33)$$

$$C''' = \frac{A z}{(T')^2} = \frac{g}{2}\left(\frac{e^{n} - 1 - n}{(e^{\frac{1}{2}n} - 1)^2}\right) \qquad (35)$$

The use of these secondary functions will appear later. They are connected with the elements of a trajectory by the following relations easily established for a given muzzle velocity:

$$A'' = \frac{y_0}{C \tan\varphi} \qquad (35)$$

$$B'' = 10\, v\left(\frac{\sin 2\,\omega}{C}\right)^{\frac{1}{2}} \qquad (36)$$

$$C' = \frac{\sin 2\,\varphi}{X} \qquad (37)$$

$$C'' = \frac{\sin\varphi}{T} \qquad (38)$$

$$C''' = \frac{2\,X \tan\varphi}{T^2} \qquad (39)$$

Other secondary functions will be given further on, in the solution of some of the minor problems; but it was not considered worth while to tabulate them an account of the little importance in practical gunnery of the problems in which the occur, and also because these problems can be solved when desired by inspection and trial as explained in Problem Thirteen.

Table 1 is a single-entry table computed for a muzzle velocity of 800 f. s. (240 m. s.). Its argument is the quotient of the horizontal range divided by the ballistic coefficient, designated by z. Of the eleven tabulated secondary functions, A'', B', B'' and C''' are independent of V and can therefore be used for all muzzle velocities within the range of the quadratic law of resistance. The remainihg seven functions depend upon V as well as z; but, as is easily seen from equations (24), (25), (26), (27), (28), (32) and (33), if these functions are tabulated for a muzzle velocity of 800 f. s. there values for any muzzle velocity less than this can be found by simply multiplying them by the ratio (or the square of the ratio) of the two muzzle velocities. By this principle not only is the labor of double interpolation (that is, interpolation with reference to both V and z), avoided, but the muzzle velocity is introduced into the equations thereby giving greater elegance and accuracy to the solutions of the problems.

By introducing V as just explained equations (12), (14), (15), (16), (17), (37) and (38) become, respectively, for *any muzzle velocity*,

$$u = v \sec \varphi \cos \omega \left(\frac{800}{V}\right) \tag{40}$$

$$\sin 2\varphi = A\,C \left(\frac{800}{V}\right)^2 \tag{41}$$

$$\tan \omega = \frac{BC}{2\cos^2 \varphi} \left(\frac{800}{V}\right)^2 \tag{42}$$

$$T = \frac{C\,T'}{\cos \varphi} \left(\frac{800}{V}\right) \tag{43}$$

$$\sin 2\omega = B\,C \left(\frac{800}{V}\right)^2 \tag{44}$$

$$C' = \frac{\sin 2\varphi}{X} \left(\frac{V}{800}\right)^2 \tag{45}$$

$$C'' = \frac{\sin \varphi}{T} \left(\frac{V}{800}\right) \tag{46}$$

If we consider these seven quantities (which may be regarded as the primary elements of a horizontal trajectory), namely, C, V, X, φ, ω, v, T, it will be found that if any three of them be known, the remaining four can be computed by means of the formulas here given, in connection with Table 1. There may therefore be thirty-five cases involving only primary elements. These are not all of equal importance and some of the cases may be of little use in gunnery. But for the sake of completeness they are all solved in the problems following, both for English and metric units, and illustrated by numerical examples.

PROBLEM ONE.

Given: C, V, X. Required: φ, ω, v, T.

Solution: Compute (see equation (13)),

$$z = \frac{X}{C}$$

and with this take u, A, $\log B'$ and T' from Table 1. Then by equations (41), (18, (40) and (43), in the order named, we have

$$\sin 2\varphi = A\,C\left(\frac{800}{V}\right)^2$$

$$\tan \omega = B' \tan \varphi$$

$$v = u \cos \varphi \sec \omega \left(\frac{V}{800}\right)$$

$$T = C\,T' \sec \varphi \left(\frac{800}{V}\right)$$

Example 1. $C = 2$, $V = 700$ f. s., $X = 1000$ yards.

We first find

$$z = \frac{X}{C} = \frac{3 \times 1000}{2} = 1500$$

and with this take from Table 1, $u = 745.9$, $A = .07902$, $\log B' = 0.02031$, $T' = 1.943$. Whence by the formulas last given, $\varphi = 5^\circ\ 57'\ 20''$, $\omega = 6^\circ\ 14'$, $v = 653$ f. s., $T = 4.465$ seconds.

For the same horizontal range and projectile, setting aside atmospheric conditions, A and C are constant in the expression

$$\sin 2\varphi = A\,C\left(\frac{800}{V}\right)^2$$

and therefore for two corresponding values of φ and V, distinguished by subscripts, we have by division

$$\frac{\sin 2\varphi}{\sin 2\varphi_1} = \left(\frac{V_1}{V}\right)^2$$

Similarly, from equation (36)

$$\frac{\sin 2\omega}{\sin 2\omega_1} = \left(\frac{v_1}{v}\right)^2$$

That is, for the same horizontal range and ballistic coefficient, the sines of twice the angles of departure are inversely as the squares of the corresponding muzzle velocities; and the sines of twice the angles of fall are inversely as the squares of the corresponding striking velocities.

From equation (18) we have, for the same conditions as before,

$$\frac{\tan \varphi}{\tan \omega} = \frac{\tan \varphi_1}{\tan \omega_1} = \text{constant}$$

That is, for the same horizontal range and ballistic coefficient, the ratios of the tangents of the angles of departure and angles of fall are equal for all trajectories described by varying the muzzle velocity, at least within the limits of direct fire.

We also have the relations

$$\frac{\tan \varphi}{\tan \varphi_1} = \frac{\tan \omega}{\tan \omega_1} = \left(\frac{T}{T_1}\right)^2$$

That is, the tangents of the angles of departure are proportional to the tangents of the corresponding angles of fall, and to the squares of the corresponding times of flight.

Example 2. In Ex. 1 suppose X and C to remain the same as there given while V becomes 750 f. s. What will be the new values of φ, ω and T?

We have

$$\sin 2\varphi = \left(\frac{V_1}{V}\right)^2 \sin 2\varphi_1 = \left(\frac{700}{750}\right)^2 \sin 11^\circ\ 54'\ 40''$$

$$\therefore\ \varphi = 5^\circ\ 10'\ 45''$$

$$\tan \omega = \frac{\tan \varphi}{\tan \varphi_1} \tan \omega_1 = \frac{\tan 5^\circ\ 10'\ 45''}{\tan 5^\circ\ 57'\ 20''} \tan 6^\circ\ 14'$$

$$\therefore\ \omega = 5^\circ\ 25'\ 20''$$

$$T = T_1 \left(\frac{\tan \varphi}{\tan \varphi_1}\right)^{\frac{1}{2}} = 5.152 \left(\frac{\tan 5^\circ\ 10'\ 45''}{\tan 5^\circ\ 57'\ 20''}\right)^{\frac{1}{2}}$$

$$\therefore\ T = 4.802 \text{ seconds.}$$

Example 3. Let X and C remain the same as before and φ be changed to 10°. What will be the muzzle velocity?

Answer.

$$V = 700 \left(\frac{\sin 11^\circ\ 54'\ 40''}{\sin 20^\circ}\right)^{\frac{1}{2}} = 543.78 \text{ f. s.}$$

It will be seen from these examples that if we have a range table for curved fire for a muzzle velocity V, we can, without recourse to any ballistic table, compute another range table having a different muzzle velocity.

Solution for metric units.

The methods developed in this paper for the solution of problems in curved fire are easily adapted to any units of length and weight. For the metric system the range and muzzle velocity are given in metres, the diameter of the projectile in centimetres and its weight in kilograms, while the coefficient of reduction, factor for atmospheric conditions and integrating factor are the same for all units. By a proper use of the factor for converting units of the metric to those of the English system the expression for C becomes

$$C = [1.15299]\, f \frac{\delta_1}{\delta} \frac{w}{c\, d^2\, \beta}$$

Also

$$z = [0.51599]\, \frac{X}{C}$$

$$\sin 2\varphi = [4.77420]\, \frac{A\, C}{V^2}$$

$$\tan \omega = B' \tan \varphi$$

$$v = [7.09691 - 10]\; V u \cos \varphi \sec \omega$$

$$T = [2.38710]\, \frac{C\, T'}{V \cos \varphi}$$

Example 4. $d = 24$ cm, $w = 144$ kg, $V = 200$ m. s., $X = 3250$ m. From Table 2 we find for the given range and for an angle of departure in the neighborhood of 30°, $\beta = 1.06$. Therefore, omitting from the calculation of C the altitude and atmospheric factors and making c unity, the work is as follows :—

Const. log $= 1.15299$
log $w = 2.15836$
a. c. log $d^2 = 7.23958$
a. c. log $\beta = 9.97469$

log $C = 0.52562$

Const. log $= 0.51599$
log $X = 3.51188$
a. c. log $C = 9.47438$

log $z = 3.50225$
$z = 3178.7$

$\therefore u = 689.9$
$A = .17681$
log $B' = 0.04300$
$T' = 4.284$

log $V = 2.30103$
log $u = 2.83879$
log cos $\varphi = 9.93344$
log sec $\omega = 0.07873$
a. c. log 800 $= 7.09691$

log $v = 2.24890$
$v = 177.4$ m. s.

Const. log $= 4.77420$
log $A = 9.24751$
log $C = 0.52562$
a. c. log $V^2 = 5.39794$

log sin $2\varphi = 9.94527$
$2\varphi = 61^\circ\, 50'\, 10''$
$\varphi = 30^\circ\, 55'\, 5''$

log tan $\varphi = 9.77734$
log $B' = 0.04300$

log tan $\omega = 9.82034$
$\omega = 33^\circ\, 28'$

Const. log $= 2.38710$
log $C = 0.52562$
log $T' = 0.63185$
a. c. log $V = 7.69897$
log sec $\varphi = 0.06656$

log $T = 1.31010$
$T = 20.42$ sec.

PROBLEM TWO.

Given: C, V, φ. Required: X, ω, v, T.

Solution: Compute (see equation (41)),

$$A = \frac{\sin 2\varphi}{C}\left(\frac{V}{800}\right)^2$$

and with this take z, u, log B', T' from Table 1. We then have (see equation (13)),

$$X = Cz$$

and ω, v, T as in Problem One.

Example 1. $C = \frac{1}{10}$, $V = 200$ f. s., $\varphi = 10°$.

Substituting these numbers in the above expression for A we find $A = .21376$, and then from Table 1 by interpolation,

$$z = 3700 + \frac{439 \times 100}{637} = 3768.9$$

$$u = 672.9 - .689 \times 3.2 = 670.7$$

$$\log B' = 0.05005 + .689 \times .00135 = 0.05098$$

$$T' = 5.049 + .689 \times .149 = 5.152$$

Therefore $X = 376.89$, $\omega = 11° 13'$, $v = 168.35$ f.s., $T = 2.093$ sec.

Example 2. The official range table* for the army magazine rifle gives $\varphi = 1° 40' 35''$ for a range of 1000 yards. Suppose we wish to practice with the sight set for this range, but with a reduced charge giving a muzzle velocity of 400 f. s. How far off should the target be placed to simulate the 1000-yard range? Take log $C = 9.46553 - 10$ which results from making $c = 1.197$.

We find $A = .050054$ and $z = 966.2$. Therefore $X = 282.23$ ft.

With the sight set for 500 yards, 1500 yards, or 2000 yards, we find in a similar manner $X = 93.1$ feet, 571.2 feet, and 970.5 feet, respectively.

Solution for metric units.

Compute C as in Problem One and then A by the equation

$$A = [5.22580 - 10]\frac{V^2 \sin 2\varphi}{C}$$

and with this take z, u, log B', T' from Table 1. We then have

$$X = [9.48401 - 10]\, Cz$$

and ω, v, T as in Problem One.

Example 3. $d = 24$ cm, $w = 144$ kg, $V = 200$ m. s., $\varphi = 30°$.

The computations may be arranged as follows: (For the value of log C see Problem One).

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* G. O. 36, A. G. O., June 11, 1897, page 89.

$$\begin{aligned}
\text{Const. log} &= 5.22580\\
\log V^2 &= 4.60206\\
\log \sin 2\,\varphi &= 9.93753\\
\text{a. c. } \log C &= 9.47438\\
\hline
\log A &= 9.23977\\
A &= .17369
\end{aligned}$$

Therefore by interpolation as in Problem One,

$$\begin{aligned}
z &= 3127.7\\
u &= 691.1\\
\log B' &= 0.04231\\
T' &= 4.210
\end{aligned}$$

$$\begin{aligned}
\text{Const. log} &= 9.48401\\
\log C &= 0.52562\\
\log z &= 3.49523\\
\hline
\log X &= 3.50486\\
X &= 3198 \text{ m.}
\end{aligned}$$

The other answers are:

$$\begin{aligned}
\omega &= 32^\circ\ 28'\ 30''\\
v &= 177.4 \text{ m. s.}\\
T &= 19.9 \text{ seconds.}
\end{aligned}$$

Example 4. $d = 21$ cm, $w = 91$ kg, $V = 204.1$ m. s., $\varphi = 45^\circ$, to compute the range. For this we have $\beta = 1.17$ and $c = 1\frac{1}{9}$. Therefore (see Problem One),

$$\log C = 0.35364.$$

Answer. $X = 3588$ m. The mean of five shots fired at Meppen with the above data was 3611 m.*

PROBLEM THREE.

Given: C, X, φ. Required: V, ω, v, T.

Solution: Compute

$$z = \frac{X}{C}$$

and with this take u, A, $\log B'$, T' from Table 1. Then

$$V = 800 \left(\frac{A\ C}{\sin 2\ \varphi}\right)^{\frac{1}{2}}$$

and ω, v, T as in Problem One.

Example 1. $C = 10$, $X = 100$ ft., $\varphi = 5^\circ$.

We find $z = 10$, and from Table 1, $u = 799.63$, $A = 0.000504$, $\log B' = 0.000135$, $T' = .0125$. Therefore $V = 136.3$ f. s., $\omega = 5^\circ\ 00'\ 6''$, $v = 136.3$ f. s., $T = 0.7337$ seconds.

These results are practically the same as those given by the parabolic theory, as they should be for so heavy a projectile and low velocity. The example is given to illustrate the precision of our method.

Example 2. $C = 2$, $X = 1600$ yards, $\varphi = 10^\circ$.

Answers. $V = 697.95$ f. s., $\omega = 10^\circ\ 46'\ 30''$, $v = 625.4$ f. s., $T = 7.391$ seconds.

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* Krupp's *Expériences de tir*, No. 31, January 19, 1882.

Example 3. Suppose that in gallery practice with the army magazine rifle, the distance to the target is 50 feet, and we set the sight for a range of 1000 yards. What should be the muzzle velocity in order to hit the target at the level of the muzzle?

We have $\varphi = 1^\circ\ 40'\ 35''$, $X = 50$ ft., $\log C = 9.46553$. Therefore $z = 171.18$ and $A = .00865$. Whence $V = 166.3$ f. s.

If we place the target 100 yards from the gun we shall find in the same way, that when the sight is set either for 500 yards, 1000 yards, 1500 yards, or 2000 yards, the following corresponding muzzle velocities will be required, namely: 726.0 f. s., 412.8 f. s., 285.6 f. s., and 214.3 f. s. Thus with suitable cartridges, easily provided, the 100-yard range (or any other convenient distance), can be used for the preliminary practice in sighting for all ranges. It is believed that the process indicated in this example would give to the rather tiresome sighting drill much of the emulation and interest attaching to actual target practice.

Application to Golf.

Example 4. What initial velocity must be given to a golf ball to send it 150 yards before striking the ground (supposed to be horizontal), with an angle of rise (or departure) of 10°?

The weight of a *new* Silvertown golf ball is 41.877 grammes (a little less than $1\frac{1}{2}$ ounces), and its diameter is $1\frac{1}{16}$ inches. The theoretical value of c is 1.391, which in the absence of experiment, we will adopt. This gives $\log C = 8.36749 - 10$.

We now have $z = X / C = 19307$,—which is below the limit of our table. To work the example it will be necessary to compute A by equation (25), in which

$$\log \frac{g}{A_1 V^2} = 0.0312446,$$

and

$$n = [5.9709214 - 10]\ z.$$

We find $A = 1.95110$ and then $V = 291.7$ f. s.

The following calculations have been made in a similar manner:

	$X = 150$ yards.			$X = 200$ yards.	
φ	V	V In vacuo	φ	V	V In vacuo
10°	291.7 f.s.	205.7 f.s.	10°	387.1 f.s.	237.5 f.s.
20	215.8	150.0	20	288.2	173.3
30	187.8	129.3	30	252.0	149.3
40	181.6	121.2	40	247.2	140.0
45	183.6	120.3	50	252.1	138.9

For these comparatively low velocities the parabolic theory (motion in *vacuo*) would give approximately correct results for artillery projectiles, but not for a golf ball, as a comparison of these values of V shows.

They also illustrate the fact, already well known, that the angle of departure giving the maximum range in air is less than 45°.

It would seem from these, and from other similar calculations, that for a long drive, where the ball strikes on ground that precludes rolling, the angle of departure should be about 42°.

Solution for metric units.

Compute z as in Problem One and with this take u, A, $\log B'$, T' from Table 1. Then

$$V = [2.38710] \left(\frac{A\,C}{\sin 2\varphi}\right)^{\frac{1}{2}}$$

and ω, v, T as in Problem One.

Example 5. $d = 24$ cm, $w = 144$ kg, $\varphi = 35°$, $X = 4000$ m.

From Table 2 we find $\beta = 1.08$, whence $\log C = 0.51751$ and $z = 3986.0$. From Table 1 we now take $u = 663.9$, $A = .22770$, $\log B' = 0.05391$, $T' = 5.477$. From these we find $V = 217.8$ m. s., $\omega = 38° \; 24'$, $v = 188.9$ m. s., $T = 24.652$ seconds,—the last three computed as in Problem One.

PROBLEM FOUR.

Given: C, φ, ω. Required: V, X, v, T.

Solution: Compute (see equation (18))

$$\log B' = \log \tan \omega - \log \tan \varphi$$

and take with this z, u, A, T' from Table 1. We then have

$$X = Cz$$

and V, v, T as in Problem Three.

Example 1. $C = \frac{1}{2}$, $\varphi = 8°$, $\omega = 10°$.

We find $\log B' = 0.09852$, $z = 7302.2$, $u = 568.54$, $A = .46692$, $T' = 10.880$; and by the formulas, $V = 736.24$ f. s., $X = 1217$ yards, $v = 526.13$ f. s. and $T = 5.967$ sec.

Solution for metric units.

Compute

$$\log B' = \log \tan \omega - \log \tan \varphi$$

and with this take z, u, A, T' from Table 1. We then have

$$X = [9.48401 - 10]\, Cz$$

and V, v, T as in Problem Three. C will be computed as in Problem One.

Example 2. $d = 27$ cm, $w = 200$ kg, $\varphi = 40°$, $\omega = 43°$.

We find from Table 2, for a range in the neighborhood of 4000 m, and for an angle of departure of 40°, that $\beta = 1.11$. Therefore by Problem One $\log C = 0.54597$. We next find $\log B' = 0.04584$; and then from Table 1, $z = 3388.15$, $u = 682.8$, $A = .18976$, $T' = 4.590$. Then, as in Problem Three, we find $V = 200.6$ m. s., $X = 3630$ m., $v = 180.1$ m. s., $T = 25.49$ seconds.

PROBLEM FIVE.

Given: V, X, φ. Required: C, ω, v, T.

Solution: Compute (see equation (45)),

$$\log C' = 4.19382 + \log \sin 2\varphi + 2 \log V - \log X - 10$$

and with this take z, u, $\log B'$, T' from Table 1. We then have

$$\log C = \log X - \log z$$

and ω, v, T as in Problem One.

Example 1. $V = 800$ f. s., $X = 2000$ yards, $\varphi = 10°$.

We find

$$\log C' = 5.7599 - 10$$

and then from Table 1, by interpolation, $z = 3921.4$, $u = 665.94$, $\log B' = 0.05304$, $T' = 5.380$. Finally, by the formulas, $\log C = 0.18471$, $\omega = 11° 16'$, $v = 668.7$ f. s., $T = 8.36$ sec.

Solution for metric units.

Compute

$$\log C' = 4.70981 + \log \sin 2\varphi + 2 \log V - \log X - 10$$

and with this take z, u, $\log B'$, T' from Table 1. We then have

$$C = [9.36300 - 10] \frac{X}{z}$$

$$T = [3.54009] \frac{C\,T'}{V \cos \varphi}$$

and ω, v as in Problem One.

Example 2. $V = 220$ m. s., $X = 3000$ m, $\varphi = 22° 6'$.

We will give the entire work as a further illustration of how to use metric data with English tables.

$$\begin{array}{rl} \text{Const. }\log &= 4.70981 \\ \log \sin 2\varphi &= 9.84334 \\ 2 \log V &= 4.68185 \\ \text{a. c. }\log X &= 6.52287 \\ \hline \log C' &= 5.76087 \end{array}$$

$$\begin{array}{rl} \text{Const. }\log &= 9.36300 \\ \log X &= 3.47712 \\ \text{a. c. }\log z &= 6.36964 \\ \hline \log C &= 9.20976 - 10 \end{array}$$

$$\therefore z = 4200 + \frac{970}{14} = 4269.3$$

$$\begin{aligned} u &= 655.2 \\ A &= .24619 \\ \log B' &= 0.05774 \\ T' &= 5.906 \end{aligned}$$

$$\begin{aligned} \text{Const. } \log &= 3.54009 \\ \log C &= 9.20977 \\ \log T' &= 0.77129 \\ \log \sec \varphi &= 0.03314 \\ \text{a. c. } \log V &= 7.65758 \\ \hline \log T &= 1.21187 \\ T &= 16.29 \text{ sec.} \end{aligned}$$

$$\begin{aligned} \log \tan \varphi &= 9.60859 \\ \log B' &= 0.05774 \\ \hline \log \tan \omega &= 9.66633 \\ \omega &= 24^{\circ}\ 53' \end{aligned}$$

$$\begin{aligned} \text{Const. } \log &= 7.09691 \\ \log V &= 2.34242 \\ \log u &= 2.81637 \\ \log \cos \varphi &= 9.96686 \\ \log \sec \omega &= 0.04231 \\ \hline \log v &= 2.26487 \\ v &= 184 \text{ m. s.} \end{aligned}$$

These results are practically the same as those deduced by Siacci's table.*

Example 3. $V = 180$ m. s., $X = 2860$ m., $\varphi = 55^{\circ}$.

We first find

$$\log C' = 5.73697 - 10$$

and then $z = 2591.3$, $u = 708.7$, $\log B' = 0.03507$, $T' = 3.444$. Therefore

$$\log C = 9.40585 - 10 = \log \left(f \frac{\delta_1}{\delta} \frac{w}{c\, d^2\, \beta} \right)$$

and $\omega = 57^{\circ}\ 8'\ 30''$, $v = 168.6$ m. s., $T = 29.45$ sec.

PROBLEM SIX.

Given: V, φ, T. Required: C, X, ω, v.

Solution: Compute (see equation (46)),

$$\log C'' = 7.09691 + \log V + \log \sin \varphi - \log T - 10$$

and with this take z, u, A, $\log B'$ from Table 1. We then have

$$C = \frac{\sin 2\varphi}{A} \left(\frac{V}{800} \right)^2$$

and X, ω, v as in Problem Two.

Example 1. $V = 300$ f. s., $\varphi = 10^{\circ}$, $T = 3$ seconds

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[*] Siacci's *Balistique Extérieure*, 1892, page 87.

We find by the given data

$$\log C'' = 8.33658 - 10$$

and from Table 1, $z = 8409$, $u = 539.9$, $A = 0.55904$, $\log B' = 0.11334$. With these values the formulas give

$$\log C = 8.93467 - 10$$

and $X = 723.5$ ft., $\omega = 12^\circ\ 54'$, $v = 204.5$ f. s.

Solution for metric units.

Compute

$$\log C'' = 7.61290 + \log V + \log \sin \varphi - \log T - 10$$

and with this take z, u, A and $\log B'$ from Table 1. We then have

$$C = [4.07281 - 10] \frac{V^2 \sin 2\varphi}{A}$$

$$X = [0.63700]\ Cz$$

and ω, v as in Problem One.

Example 2. $V = 200$ m. s., $\varphi = 30^\circ\ 55'\ 5''$, $T = 20.42$ seconds. We find

$$\log C'' = 8.31463 - 10$$

and from Table 1, $z = 3176.3$, $u = 689.6$, $A = .17667$, $\log B' = 0.04297$. Therefore

$$\log C = 9.37298 - 10$$

and the other elements as in Ex. 4 of Problem One.

PROBLEM SEVEN.

Given: V, φ, ω. Required: C, X, v, T.

Solution: Compute (see equation (18))

$$\log B' = \log \tan \omega - \log \tan \varphi$$

and with this take z, u, A, T' from Table 1. We then have

$$\log C = 4.19382 + \log \sin 2\varphi + 2 \log V - \log A - 10$$

and X, v, T as in Problem Two.

Example 1. $V = 700$ f. s., $\varphi = 10^\circ$, $\omega = 12^\circ$. We find

$$\log B' = 0.08115$$

and from Table 1, $z = 6008.2$, $u = 604.1$, $A = .36739$ and $T' = 8.672$. With these tabular values the formulas give

$$\log C = 9.85294 - 10$$

and $X = 4282.3$ ft., $v = 532.2$ f. s., $T = 7.173$ seconds.

Solution for metric units.

Compute

$$\log B' = \log \tan \omega - \log \tan \varphi$$

and with this take z, u, A, T' from Table 1. We have then

$$\log C = 4.07281 + \log \sin 2\varphi + 2 \log V - \log A - 10$$

$$X = [0.63700]\, Cz$$

and v and T as in Problem One.

Example 2. $V = 201.5$ m. s., $\varphi = 40°$, $\omega = 43°$.

We find

$$\log B' = 0.04584$$

and from Table 1, $z = 3388.15$, $u = 682{\cdot}8$, $A = .18976$ and $T' = 4.590$. Therefore

$$\log C = 9.39278 - 10$$

$$X = 3630 \text{ m.}$$

and the other elements as in Ex. 5, Problem Three.

PROBLEM EIGHT.

Given: X, φ, T. Required: C, V, ω, v.

Solution: Compute (see equation (39))

$$\log C''' = 0.30103 + \log X + \log \tan \varphi - 2 \log T$$

and with this take z, u, A, $\log B'$ from Table 1. We then have

$$\log C = \log X - \log z$$

and V, ω, v as in Problem Three.

Example 1. The six shots fired from the 8-inch pneumatic torpedo gun at Fort Lafayette, September 20, 1887, at the Schooner *Silliman* give the following data: Diameter of projectile 7.75 inches, weight of projectile including charge $137\frac{1}{6}$ lbs., mean range 5583.5 ft., angle of departure 14° 53′ 20″, mean time of flight 10.63 seconds.*

Here $X = 5583.5$ ft., $\varphi = 14° 53' 20''$, $T = 10.63$ seconds. Therefore

$$\log C''' = 1.41955$$

and from Table 1, $z = 13925$, $u = 417$, $A = 1.13497$, $\log B' = 0.18621$. With these numbers the formulas give

$$\log C = 9.60311 - 10$$

and $V = 765.8$ f, s., $\omega = 22° 12'$, $v = 416.7$ f. s.

A standard projectile of the same diameter and weight would give

$$\log C = 0.35865$$

Whence it follows that this projectile with its spiral wings for imparting rotation, and its very unsteady flight, suffered a resistance more than 5½ times that of a service projectile.

These projectiles have been very much improved since the date of the above firing. From the report of the firing of a pneumatic torpedo gun at Shoeburyness, England, in the summer of 1890, made by B. C. Batcheller, S. B.,[*] with 8 and 10-inch sub-caliber and 15-inch full caliber dummy projectiles, is taken the following data:

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* This example will be found in the *Handbook*, page 117; in Vallier's *Balistique Extérieure*, page 93; and in La Llave's *Problemas de Balistica*, page 67.

8-inch.	10-inch.
$w = 298$ lbs.	$w = 493$ lbs.
$d = 8.27$ in.	$d = 10.25$ in.
$\varphi = 15^\circ\ 8'$	$\varphi = 15^\circ\ 10'$
$X = 2931$ yds.	$X = 2298$ yds.
$T = 12.835$ sec.	$T = 11.2$ sec.

With these data Problem Eight readily gives for the 8-inch gun $V = 822$ f. s.; and for the 10-inch gun $V = 718.6$ f. s., while the coefficients of reduction are, respectively, 2.834 and 3.403.

It will be seen that the "laborious tentative process" by which this problem was originally solved is reduced by this method to a simple, direct and easy process of as many minutes as the former required hours.

Example 2. $X = 6001$ ft., $\varphi = 18^\circ$, $T = 11.8$ sec.[†]

Answers.

$\log C''' = 1.44727$, $\log C = 9.81184 - 10$, $V = 666.7$ f. s., $\omega = 23^\circ\ 22'$ and $v = 448$ f. s.

Solution for metric units.

Compute

$$\log C''' = 0.81702 + \log X + \log \tan \varphi - 2 \log T$$

and with this take z, u, A, $\log B'$ from Table 1. We then have

$$\log C = 9.36300 + \log X - \log z - 10$$

$$V = [2.96360] \left(\frac{A\ C}{\sin 2\varphi}\right)^{\frac{1}{2}}$$

and the other elements as in Problem One.

Example 3. $X = 1701.8$ m, $\varphi = 14^\circ\ 53'\ 20''$, $T = 10.63$ sec.

We find

$$\log C''' = 1.41954$$

the same as in Ex. 1, and therefore the numbers to be taken from Table 1 are the same as there given. The above formulas give

$$\log C = 8.45012 = \log \frac{w}{c\ d^2}$$

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[*] See *Journal U. S. Artillery*, whole No. 19.

[†] The data of this example are taken from *Pro. U. S. Naval Inst.*, whole No. 69, page 30.

To find the value of the coefficient of reduction we have

$$c = \frac{w}{C d^2}$$

In our example $w = 62.2$ kg. and $d = 19.68$ cm. Whence

$$c = 5.7$$

PROBLEM NINE.

Given: X, φ, ω. Required: C, V, v, T.

Solution: Compute log B' as in Problem Four, with which take z, u, A, T' from Table 1. We then have

$$\log C = \log X - \log z$$

and V, v, T as in Problem Three.

Example 1. $X = 1000$ yds.. $\varphi = 15°$, $\omega = 16°$.

We find

$$\log B' = 0.02944$$

and then $z = 2175.6$, $u = 722.6$, $A = 0.11714$, $T' = 2.863$. Whence

$$\log C = 0.13954$$

and $V = 454.7$ f. s., $v = 412.7$ f. s. and $T = 7.191$ sec.

Example 2. $X = 10$ ft., φ 10°, $\omega = 11°$.

Answers. $\log C = 7.50461 - 10$, $V = 32.236$ f.s., $v = 27.937$ f.s., $T = 0.339$ sec.

Solution for metric units.

Compute log B' as in Problem Four, with which take z, u, A, T' from Table 1. We then have log C, V, v and T as in Problem Eight.

Example 3. $X = 2860$ m, $\varphi = 55°$, $\omega = 57° \, 8' \, 30''$.

We find

$$\log B' = 0.03506$$

$A = .14140$, and the other numbers as in Ex. 2, Problem Five. Whence

$$\log C = 9.40588 = \frac{w}{c\, d^2 \beta}$$

PROBLEM TEN.

Given: φ, ω, T. Required: C, V, X, v.

Solution: Compute log B' as in Problem Four, with which take z, u, A, T' from Table 1. We then have by combining equations (41) and (43),

$\log C = 9.69897 + \log A - \log \tan \varphi + 2 \log T - 2 \log T' - 10$
and V, X, v as in Problems Two and Three.

Example 1. $\varphi = 15°$, $\omega = 16°$, $T = 12$ seconds.

The values of log B', z, u, A and T' are the same as those in Ex. 1 of Problem Nine. Whence

$$\log C = 0.58434$$

and $V = 758.8$ f. s., $X = 8354.5$ ft., $v = 688.7$ f. s.

Solution for metric units.

Compute log B' as in Problem Four, with which take z, u, A, T' from Table 1. We then have

$$\log C = 8.54598 + \log A - \log \tan \varphi + 2 \log T - 2 \log T' - 10$$

and V, X, v as in Problems Eight and Seven.

Example 2. $\varphi = 15°$, $\omega = 16°$, $T = 12$ seconds.

The tabular numbers are the same as in Ex. 1 for English units. Whence

$$\log C = 9.43135$$

and $V = 231.3$ m. s., $X = 2546$ m, $u = 209.9$ m. s.

PROBLEM ELEVEN.

Given: φ, ω, v. Required: C, V, X, T.

Solution: Compute log B' as in Problem Four, with which take z, u, A, T' from Table 1. We then have

$$V = \frac{800\,v}{u} \sec \varphi \cos \omega$$

and C, X, T as in Problems Two and Six.

Example 1. $\varphi = 5°$, $\omega = 7° 30'$, $v = 50$ f. s.

We find

$$\log B' = 0.17748$$

and by interpolation from Table 1, $z = 13257.25$, $u = 430.3$, $A = 1.05330$, $T' = 22.959$. Whence $V = 92.52$ f. s., $\log C = 7.34338 - 10$, $X = 29.23$ ft., $T = 0.4394$ sec.

Solution for metric units.

The computation of log B' and V is the same for metric as for English units. Then compute C and X as in Problem Six, and T as in Problem Five.

Example 2. $\varphi = 5°$, $\omega = 7° 30'$, $v = 15.24$ m. s.

All the tabular numbers are the same as in Ex. 1. Substituting these numbers in the formulas indicated we have

$$V = 28.2 \text{ m. s.}$$

$\log C = 6.19039 - 10$, $X = 8.9$ m., $T = 0.4394$ sec.

PROBLEM TWELVE.

Given: C, X, ω. Required: V, φ, v, T.

Solution: Compute

$$z = \frac{X}{C}$$

with which take u, A, $\log B'$, T' from Table 1. We then have

$$\tan \varphi = \frac{\tan \omega}{B'}$$

and V, v, T as in Problems One and Three.

Example 1. $C = 2$, $X = 1000$ yds., $\omega = 15°$.

We find $z = 1500$, $u = 745.9$, $A = .07902$, $\log B' = 0.02031$, and $T' = 1.943$. Therefore by the formulas, taken in order of computation, $\varphi = 14° 20' 40''$, $V = 459.0$ f. s., $v = 429.3$ f. s., $T = 6.991$ sec.

Example 2. It is desired to open a searching fire upon the enemy (protected by cover from direct fire), with shrapnel from a battery of 3.2-inch field guns, at 2000 yards. The angle of descent should be 20° to be effective. What must be the muzzle velocity and angle of elevation—also striking velocity and time of flight?

Here $X = 6000$, $\omega = 20°$, and $\log C = 0.00816$. We find $z = 5888.3$, $A = .35859$, $\log B' = 0.07954$, $T' = 8.474$ and $u = 607.4$. Therefore by the formulas, $\varphi = 16° 51' 30''$, $V = 649.01$ f. s., $v = 501.9$ f. s., $T = 11.121$ sec.

If it were considered desirable to increase the angle of fall without changing the position of the battery, it could be done only by increasing the angle of elevation and decreasing the muzzle velocity. Suppose we increase the angle of departure to 20°, what should the muzzle velocity be?

Answer.

$$V = 649.01 \left(\frac{\sin 33° 43'}{\sin 40°} \right)^{\frac{1}{2}} = 603.2 \text{ f. s.}$$

Solution for metric units.

Compute z and take out the tabular numbers as in Problem One. Then compute φ as for English units, and V, v, T by Problem Three.

Example 3. $d = 24$ cm, $w = 144$ kg, $c\beta = 1.1574$, $X = 2000$ m, $\varphi = 45°$.

We find

$$\log C = 0.48744$$

$z = 2135.9$, $u = 724.0$, $A = .11485$, $\log B' = 0.02890$, $T' = 2.809$. Therefore $\varphi = 43° 5' 40''$, $V = 145$ m. s., $v = 135.5$ m. s., $T = 19.87$ sec.

PROBLEM THIRTEEN.

Given: C, φ, T. Required: V, X, ω, v.

Solution: By combining equations (14), (16), (28) and (33) we obtain the equalities

$$\frac{C \tan \varphi}{T^2} = \frac{A}{2\ T'^2} = \frac{C''}{T'} = \frac{g}{4z}\left(\frac{e^n - 1 - n}{(e^{\frac{1}{2}n} - 1)^2}\right)$$

The first of these terms is a function of the given elements C, φ and T, and the last term shows that the value of this function depends on z only. In computing Table 1, therefore, it would have been easy to prepare an additional column of this function whereby z could be taken directly from the table in any particular case. But the little importance of the problem does not warrant such an extension of the table; and besides z can be found, when required, without much additional labor by "trial and error," as follows: Compute

$$\log C'' - \log T'$$

by means of the given data. Then, with the help of a table of logarithms, a near value of z as tabulated can be found by simple inspection of Table 1. A nearer value can be computed by another trial.*

Example 1. $C = 3$, $\varphi = 5°$, $T = 3$ sec.

We find

$$\log C'' - \log T' = 8.46483 - 10$$

which is satisfied when $z = 546.18$. Therefore $u = 779.9$, $A = .02792$, $\log B' = 0.00740$. Whence by Problems Two and Three $V = 555.6$ f. s., $X = 1638.5$ ft., $\omega = 5° 5'$, $v = 541.7$ f. s.

Solution for metric units.

Compute $\log C$ as in Problem One for metric units and then find

$$\log C'' - \log T'$$

as above for English units, and take z, u, A, $\log B'$ from Table 1 as already explained. Then compute V by Problem Three, X by Problem Four and ω, v by Problem One.

Example 2. $d = 24$ cm, $w = 144$ kg, $\varphi = 30°$, $T = 19.93$ sec.

From Table 2 we find $\beta = 1.06$, and then by Problem One

$$\log C = 0.52562$$

Whence by the given data,

$$\log C'' - \log T' = 7.68805 - 10$$

which is satisfied when $z = 3142.2$. The remaining elements can now be computed as already explained.

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* See *Handbook*, pages 62-64 for an illustration of the method by "trial and error."

PROBLEM FOURTEEN.

Given: C, ω, T. Required: V, X, φ, v.

Solution: By combining equations (16), (18), (26) and (28) we obtain the equalities

$$\frac{2\,C\tan\omega}{T^2} = \frac{B}{T'^2} = \frac{g}{2\,z}\left(\frac{1-(1-n)\,e^{n}}{(e^{\frac{1}{2}n}-1)^2}\right)$$

by means of which we can compute

$$\log B - 2\log T'$$

with the given data, and find z from Table 1 by inspection and trial as explained in Problem Thirteen. Then take out u, A, $\log B'$ and compute

$$\tan\varphi = \frac{\tan\omega}{B'}$$

and the other elements as in Problems Two and Three.

Example 1. $C = 2$, $\omega = 20°$, $T = 10$ seconds.

We find

$$\log B - 2\log T' = 8.16313 - 10$$

and by trial that it is satisfied when $z = 2289$. Therefore $u = 718.8$, $A = .12369$, $\log B' = 0.03098$. Whence $X = 4578$ ft., $\varphi = 18° 43'$, $V = 510.4$ f. s., $v = 462.2$ f. s.

Solution for metric units.

Proceed as in Problem Thirteen.

PROBLEM FIFTEEN.

Given: C, ω, v. Required: V, X, φ, T.

Solution: Compute (see equations (12), (15) and (31))

$$B'' = 10\,v\left(\frac{\sin 2\,\omega}{C}\right)^{\frac{1}{2}}$$

with which take z, u, $\log B'$, T' from Table 1 and then compute the required elements as in Problems One, Two, Eleven and Fourteen.

Example 1. $C = 1$, $\omega = 15°$, $v = 200$ f. s.

We get from the given data

$$B'' = 1414.2$$

from which we find $z = 635.2$, $u = 776.6$, $\log B' = 0.00861$, $T' = 0.806$. Whence $V = 206$ f. s., $X = 635.2$ ft., $\varphi = 14° 43'$ and $T = 3.236$ seconds.

Example 2. "An escarp had to be breached at the siege of

Strasburg by a gun, equivalent to an 8-inch howitzer of 70 cwt., on the same level. From information received from a spy, the ditch was known to be about 50 feet wide; in consequence of which the necessary angle of descent was calculated to be 14°. The howitzer was using common shell and delay action fuze, and the Engineers required that the striking velocity was not to fall short of 600 f. s.

Find the least requisite distance of the howitzer from the escarp, the necessary muzzle velocity and angle of departure."*

Here $w = 180$, $d = 8$, $c = \frac{10}{8}$, $\log C = 0.40333$, $v = 600$ f. s., $\omega = 14°$.

From the given data we get

$$B'' = 2584$$

and then from Table 1, $z = 2224$, $u = 721.0$, $\log B' = 0.03010$, $T' = 2.930$. Therefore $V = 663$ f. s., $X = 1876$ yds., $\varphi = 13° 6'$, $T = 9.18$ sec.

Example 3. Suppose (see Ex. 2) that it was found necessary to increase the striking velocity to 700 f. s.

We now have $B'' = 3014.7$, $z = 3108.6$, $u = 691.7$, $\log B' = 0.04206$. Therefore $V = 805$ f. s., $X = 2623$ yds., and $\varphi = 12° 45'$.

The muzzle velocity would therefore have to be increased 142 f. s. and the guns retired 747 yards. This supposes the ground to be level. The case where the guns and the escarp to be breached are at different levels will be considered later.

Example 4. The guns are to be mounted in the second parallel which is 800 yards from the escarp. If the striking velocity is to be 600 f. s. what will be the muzzle velocity, angle of projection and striking angle?

Here $\log C = 0.40333$, $X = 2400$ ft., $v = 600$ f. s. to find V, φ and ω. Solved by Problem Twenty-one. *Answers.*

$V = 627$ f. s.
$\varphi = 5° 50'$
$\omega = 6° 1'$

Example 5. If ω is 14° what will v be, the range remaining the same as in Ex. 4? Also compute V and φ.

Here we have given C, X and ω. Solved by Problem Twelve. *Answers.*

$V = 417$ f. s.
$\varphi = 13° 36' 30''$
$v = 399.6$ f. s.

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* *Text Book of Gunnery.* London, 1897. Page 269.

Solution for metric units.

Compute C as in Problem One, and then B'' by the equation

$$B = [1.51599]\, v \left(\frac{\sin 2\omega}{C}\right)^{\frac{1}{2}}$$

with which take z, u, log B', T' from Table 1. Then compute φ by Problem Twelve, V by Problem Eleven, X by Problem Two and T by Problem One.

Example 6. $d = 27$ cm, $w = 200$ kg, $\omega = 7°$, $v = 200$ m. s.

We first find

$$\log C = 0.59130$$

and then

$$B'' = 1633.8$$

by means of which we get from the table, $z = 853$, $u = 768.8$, $\log B' = 0.01155$, $T' = 1.088$. Therefore $\varphi = 6° 49'$, $V = 208\ 1$ m. s., $X = 1015$ m, $T = 5.01$ sec.

PROBLEM SIXTEEN.

Given: X, T, ω. Required: C, V, φ, v.

Solution: By combining equations (13), (26), (28), (42) and (43) we obtain

$$\frac{2 X \tan \omega}{T^2} = \frac{B z}{T'^2} = \frac{g}{2}\left(\frac{1 - (1 - n)\, e^{u}}{(e^{\frac{1}{2} u} - 1)^2}\right)$$

by means of which, with the given data, we can compute

$$\log B + \log z - 2 \log T'$$

and then find z from Table 1 by inspection and trial. Next take out u, A, log B' and compute the required elements as in Problems Three, Five and Fourteen.

Example 1. $X = 3000$ ft., $\omega = 16°$, $T = 7$ seconds.

From the given data we find

$$\log B + \log z - 2 \log T' = 1.54545$$

which by trial is found to be satisfied when

$$z = 5531.3$$

Whence, $\log C = 9.73429$, $V = 503.2$ f. s., $\varphi = 13° 34' 20''$, $v = 391.0$ f. s.

Solution for metric units.

Compute

$$\frac{B z}{T'^2} = [0.81702] \frac{X \tan \omega}{T^2}$$

and proceed as with English units.

PROBLEM SEVENTEEN.

Given: X, ω, v. Required: C, V, φ, T.

Solution: By combining equations (13), (24), (26), (40) and (42), eliminating all the elements except those given by the problem, we have

$$\frac{v^2 \sin 2\omega}{X} = \frac{B u^2}{z} = 2 g \left(\frac{1 - (1 - n) e^n}{n^2 e^n} \right)$$

by means of which, using the giving data, we can compute

$$\log B + 2 \log u - \log z$$

and find z from Table 1 by inspection and trial. Then proceed as in previous problems.

Example. $X = 6000$ ft., $\omega = 15°$, $v = 600$ f. s.

We find

$$\log B + 2 \log u - \log z = 1.47712$$

which is satisfied when

$$z = 2270$$

Therefore $\log C = 0.42212$, $\varphi = 14° \, 1'$, $V = 664.2$ f. s., $T = 9.823$ sec.

Solution for metric units.

Compute

$$\frac{B u^2}{z} = [0.51599] \frac{v^2 \sin 2\omega}{X}$$

and procced as already explained.

PROBLEM EIGHTEEN.

Given: ω, v, T. Required: C, V, X, φ.

Solution: The combination of equations (24), (26), (28), (40), (42) and (43) gives

$$\frac{2 v \sin \omega}{T} = \frac{B u}{T'} = g \left(\frac{1 - (1 - n) e^n}{n \, e^{\frac{1}{2} n} (e^{\frac{1}{2}} - 1)} \right)$$

by means of which, and the given data, compute

$$\log B + \log u - \log T'$$

and proceed as in Problem Thirteen.

Example. $\omega = 20°$, $v = 100$ f. s., $T = 2.2$ sec.

We find

$$\log B + \log u - \log T' = 1.49266$$

which is satisfied when

$$z = 4788$$

Therefore $\log C = 8.68293 - 10$, $V = 123.18$ f. s., $X = 231$ ft., and $\varphi = 17° \, 24' \, 35''$.

Solution for metric units.

Compute

$$\frac{Bu}{T'} = [0.81702]\frac{v \sin \omega}{T}$$

and proceed as already explained.

PROBLEM NINETEEN.

Given: C, V, ω. Required: X, φ, v, T.

Solution: For a first approximation assume

$$\cos \varphi = \cos \omega$$

and compute (Eq. (42))

$$B = \frac{\sin 2\omega}{C}\left(\frac{V}{800}\right)^2$$

with which take z, u, A, T' from Table 1, and then compute X, φ, v, T as in Problems One and Two. Repeat if necessary making

$$B = \frac{2 \cos^2 \varphi \tan \omega}{C}\left(\frac{V}{800}\right)^2$$

which is its true value.

Example. $C = 3$, $V = 650$, $\omega = 5°$.

We find by computation

$$B = .0382$$

and then from Table 1, $z = 727.3$, $u = 773.3$, $A = .03739$, $T' = 0.925$. Whence $X = 2181.9$ ft., $\varphi = 4° 53'$, $v = 628.3$ f. s., $T = 3.428$ sec. These results are practically correct, but when φ and ω differ considerably from each other it will be necessary to repeat where great accuracy is desired.

Solution for metric units.

Compute log C as in Problem One, and B by the approximate equation

$$B = [5.22580 - 10]\frac{V^2 \sin 2\omega}{C}$$

Repeat as explained above.

PROBLEM TWENTY.

Given: C, V, v. Required: X, φ, ω, T.

Solution: Assuming for a first approximation

$$\cos \varphi = \cos \omega$$

compute

$$u = v\left(\frac{800}{V}\right)$$

with which take z, A, $\log B'$, T' from Table 1, and then compute the required elements as in Problems One and Two. If necessary u can be re-computed by the complete formula

$$u = v \sec \varphi \cos \omega \left(\frac{800}{V}\right)$$

Example. $C = 1$, $V = 800$ f. s., $v = 732$ f. s.

We find by the approximate formula

$$u = 732$$

which corresponds in Table 1 to $z = 1900$, $A = .10139$, $\log B' = 0.02572$, $T' = 2.484$. Whence $X = 1900$ ft., $\varphi = 2° 54' 30''$, $\omega = 3° 5'$ and $T = 2.487$ sec. These results need no correction.

Solution for metric units.

Same as for English units.

PROBLEM TWENTY-ONE.

Given: C, X, v. Required: V, φ, ω, T.

Solution: Compute

$$z = \frac{X}{C}$$

with which take u, A, $\log B'$, T' from Table 1. Then assuming for a first approximation

$$\cos \varphi = \cos \omega$$

compute

$$V = \frac{800\, v}{u}$$

and then φ, ω, T as in Problems One and Two. Repeat when necessary by making

$$V = \frac{800\, v}{u} \sec \varphi \cos \omega$$

which is its true value.

Example 1. $C = 1$, $X = 3000$ ft., $v = 600$ f. s.

Answers. $V = 690$ f. s., $\varphi = 6° 26'$, $\omega = 7° 3'$, $T = 4.66$ sec.

Solution for metric units.

Compute z as in Problem One and then V as above for English units.

Example 2. $d = 24$ cm, $w = 144$ kg, $X = 3250$ m, $v = 177.4$ m.

We have already found (see Ex. 4, Problem One)

$$u = 689.9,\ A = .17681,\ \log B' = 0.04300,\ T' = 4.284.$$

The successive approximations in this case are the following:

V	φ	ω
205.7 m.	28° 14′	30° 40′
200.8	30 30	33 2
200.1	30 51	33 24
200	30 55	33 28

The last set of values are correct.

PROBLEM TWENTY-TWO.

Given: V, X, v. Required: C, φ, ω, T.

Solution: Proceed as in Problem Twenty, computing the required elements as in Problems One, Two and Five.

Example. $V = 800$ f. s., $X = 4000$ ft., $v = 692$ f. s.

Answers. $\log C = 0.11070$, $\varphi = 6° 25'$, $\omega = 7° 3'$, $T = 5.41$ sec.

Solution for metric units.

Proceed as for English units.

PROBLEM TWENTY-THREE.

Given: V, φ, v. Required: C, X, ω, T.

Solution: Proceed as in Problem Twenty, computing the required elements as in Problems Two and Six.

Example. $V = 200$ f. s., $\varphi = 2°$, $v = 116.25$ f. s.

Answers. $\log C = 7.70198 - 10$, $X = 58.4$ ft., $\omega = 2° 52'$, $T = 0.39$ sec.

Solution for metric units.

Proceed as for English units.

PROBLEM TWENTY-FOUR.

Given: V, v, T. Required: C, X, φ, ω.

Solution: Proceed as in Problem Twenty, computing φ by the equation

$$\sin \varphi = \frac{A\ T}{2\ T'} \left(\frac{800}{V}\right)^2$$

obtained by combining equations (41) and (43); and the other elements as in Problems Two and Six. Repeat with cos φ and cos ω if necessary.

Example. $V = 800$ f. s., $v = 732$ f. s., $T = 2$ sec.

Answers. $\log C = 9.90554 - 10$, $X = 1528.6$ ft., $\varphi = 2° 20'$, $\omega = 2° 29'$.

Solution for metric units.

Proceed as for English units computing sin φ by the equation

$$\sin \varphi = [4.47317] \frac{A\ T}{V^2\ T'}$$

PROBLEM TWENTY-FIVE.

Given: V, ω, v. Required: C, X, φ, T.

Solution: Proceed as in Problem Twenty, computing the required elements as in Problems Two, Six and Fourteen.

Example. $V = 800$ f. s., $\omega = 20°$, $v = 692$ f. s.

Answers. $\log C = 0.50160$, $X = 10593$ ft., $\varphi = 18° \ 10'$, $T = 15.08$ sec.

Solution for metric units.

Same as for English units.

PROBLEM TWENTY-SIX.

Given: C, φ, v. Required: V, X, ω, T.

Solution: Combining equations (40) and (41), eliminating V, we have

$$A u^2 = \frac{2 v^2 \tan \varphi \cos^2 \omega}{C} = 2 g z \left(\frac{e^n - 1 - n}{n^2 e^n}\right)$$

which, if we assume for a first approximation,

$$\cos \varphi = \cos \omega$$

becomes

$$A u^2 = \frac{v^2 \sin 2 \varphi}{C}$$

By means of this last equation find z from Table 1, by inspection and trial, and then compute the required elements by methods already given, repeating if necessary by using the complete expression for $A u^2$.

Example. $C = 5$, $\varphi = 2°$, $v = 500$ f. s.

Answers. $V = 502.3$ t. s., $X = 545.7$ ft., $\omega = 2° \ 00' \ 24''$, $T = 0.685$ sec.

Solution for metric units.

The ballistic coefficient will be computed as in Problem One, and then $A u^2$ by the equation

$$A u^2 = [1.03198] \frac{v^2 \sin 2 \varphi}{C}$$

Then proceed as explained for English units.

PROBLEM TWENTY-SEVEN.

Given: V, ω, T. Required: C, X, φ, v.

Solution: Combining equations (42) and (43), eliminating C, we obtain

$$\frac{B}{T'} = \frac{2\,V \cos\varphi \tan\omega}{800\ T} = \frac{g}{V}\left(\frac{1 - (1 - n)\,e^{n}}{e^{\frac{1}{2}n} - 1}\right)$$

or, assuming

$$\cos\varphi = \cos\omega$$

we have

$$\frac{B}{T'} = \frac{2\,V \sin\omega}{800\ T}$$

with which proceed as already explained.

Example. $V = 800$ f. s., $\omega = 5°$, $T = 4$ sec.

We find

$$\log B - \log T' = 8.63927 - 10$$

which is satisfied when

$$z = 2046$$

Answers. $\log C = 0.17208$, $X = 3041$ ft., $\varphi = 4° 41'$, $v = 727$ f.s. These require no correction.

Solution for metric units.

Compute first

$$\frac{B}{T'} = [7.91393 - 10]\,\frac{V \sin\omega}{T}$$

and then proceed as already explained.

PROBLEM TWENTY-EIGHT.

Given: X, φ, v. Required: C, V, ω, T.

Solution: Combining equations (13), (40) and (41), eliminating C and V, we obtain

$$\frac{A\,u^2}{z} = \frac{2\,v^2 \tan\varphi \cos^2\omega}{X} = 2\,g\left(\frac{e^{n} - 1 - n}{n^2\,e^{n}}\right)$$

or, assuming

$$\cos\varphi = \cos\omega$$

we have

$$\frac{A\,u^2}{z} = \frac{v^2 \sin 2\,\varphi}{X}$$

to be employed as a first approximation.

Example. $X = 1600$ ft., $\varphi = 5°$, $v = 540$ f. s.

Answers. $\log C = 0.80031$, $V = 546.4$ f. s., $\omega = 5° 2' 20''$, $T = 3.73$ sec.

Solution for metric units.

Compute first

$$\frac{A\,u^2}{z} = [0.51599]\,\frac{v^2 \sin 2\,\varphi}{X}$$

and proceed as for English units.

PROBLEM TWENTY-NINE.

Given: φ, v, T. Required: C, V, X, ω.

Solution: Combining equations (40), (41) and (43), eliminating C and V, we obtain

$$\frac{Au}{T'} = \frac{2\,v \tan\varphi \cos\omega}{T} = g\left(\frac{e^{n} - 1 - n}{n\,e^{\frac{1}{2}n}\left(e^{\frac{1}{2}n} - 1\right)}\right)$$

which by the usual assumption becomes

$$\frac{Au}{T'} = \frac{2\,v \sin\varphi}{T}$$

for a first approximation.

Example. $\varphi = 5^\circ$, $v = 550$ f. s., $T = 4$ sec.

Answers. $\log C = 9.52575 - 10$, $V = 791.9$ f. s., $X = 2615$ ft., $\omega = 6^\circ\ 21'\ 30''$.

Solution for metric units.

First compute

$$\frac{Au}{T'} = [0.81612]\frac{v \sin\varphi}{T}$$

and proceed as already explained in previous problems.

PROBLEM THIRTY.

Given: V, X, ω. Required: C, φ, v, T.

Solution: Combining equations (13) and (42), eliminating C, we have

$$\frac{B}{z} = \frac{2\cos^2\varphi \tan\omega}{X}\left(\frac{V}{800}\right)^2 = \frac{2g}{V^2}\left(\frac{1-(1-n)\,e^{n}}{n^2}\right)$$

which, if we assume that

$$\cos\varphi = \cos\omega$$

becomes

$$\frac{B}{z} = \frac{\sin 2\omega}{X}\left(\frac{V}{800}\right)^2$$

for a first approximation.

Example. $V = 550$ f. s., $X = 1600$ f. s., $\omega = 5^\circ$.

We find by the approximate formula

$$\log B - \log z = 5.71009 - 10$$

which is satisfied when

$$z = 296.5$$

Therefore by Problems One and Five, $\log C = 0.73219$, $\varphi = 4^\circ\ 57'$, $v = 542$ f. s., $T = 2.95$ sec. These results need no correction.

Solution for metric units.

We have

$$\frac{B}{z} = [5.01084 - 10] \frac{V^2 \cos^2 \varphi \tan \omega}{X}$$

or, approximately,

$$\frac{B}{z} = [4.70981 - 10] \frac{V^2 \sin 2 \omega}{X}$$

PROBLEM THIRTY-ONE.

Given: C, V, T. Required: X, φ, ω, v.

Solution: We have from equation (43)

$$T' = \frac{T \cos \varphi}{C} \left(\frac{V}{800} \right)$$

For a first approximation assume

$$\cos \varphi = 1$$

and compute

$$T' = \frac{T}{C} \left(\frac{V}{800} \right)$$

with which take A from Table 1. Then compute φ by equation (41), and a new and more correct value of T' by the complete formula, with which take A again from the table. Repeat these operations until the change in T' is unimportant. The required elements can then be computed as in Problems One and Two. When φ is small but one correction of T' will be necessary.

Example. $C = 3$, $V = 800$ f. s., $T = 10$ sec.

We first find $T' = 3\frac{1}{3}$ and $\varphi = 12° 7'$. Next $T' = 3.259$ and $\varphi = 11° 49'$. Finally $T' = 3.263$, and no further correction is necessary.

Answers. $X = 7380$ ft., $\varphi = 11° 49'$, $\omega = 12° 44'$, $v = 715$ f. s.

Solution for metric units.

Compute C as in Problem One, and then T' by the formulas

$$T' = [7.61290 - 10] \frac{T V}{C}$$

and

$$T' = [7.61290 - 10 \frac{T V \cos \varphi}{C}$$

PROBLEM THIRTY-TWO.

Given: C, X, T. Required: V, φ, ω, v.

Solution: Compute (see equation (13))

$$z = \frac{X}{C}$$

and with this take u, A, log B', T' from Table 1. We then have from equation (43)

$$V = \frac{800\, C\, T' \cos\varphi}{T}$$

and the required elements as in Problem One. In computing V it will be necessary to assume for a first approximation

$$\cos\varphi = 1$$

which is accurate enough when φ is less than 5°. For greater values of φ we must repeat the operation as many times as is necessary.

Example. $C = 3$, $X = 2000$ yds., $T = 10$ sec.

We find $z = 2000$, $u = 728.6$, $A = 0.10707$, log $B' = 0.02707$, $T' = 2.621$. Omitting cos φ we find $V = 629.04$ and $\varphi = 15° 39'$. Introducing cos φ and repeating the operation four times we finally obtain $V = 600.3$ f. s. and $\varphi = 17° 23'$ which are their true values. The other elements are $\omega = 18° 26'$, $v = 550.0$ f. s.

Solution for metric units.

Compute z as in Problem One with which take u, A, log B' and T' from Table 1. We then have

$$V = [2.38710] \frac{C\, T' \cos\varphi}{T}$$

PROBLEM THIRTY-THREE.

Given: V, X, T. Required: C, φ, ω, v.

Solution: Compute (see equations (13) and (43)),

$$\frac{T'}{z} = \frac{T\, V \cos\varphi}{800\, X} = \frac{2}{V}\left(\frac{e^{\frac{1}{2}n} - 1}{n}\right)$$

assuming for a first approximation cos $\varphi = 1$. Find z from the table by inspection and trial and take out u, A, log B'. The required elements can then be computed as in Problems One and Five. Repeat the operation with cos φ as many times as may be necessary.

Example. $V = 550$ f. s., $X = 1600$ ft., $T = 3$ sec

Answers. log $C = 0.15104$, $\varphi = 5° 4' 35''$, $\omega = 5° 15' 30''$, $v = 522$ f. s.

Solution for metric units.

Same as for English units.

PROBLEM THIRTY-FOUR.

Given: C, v, T. Required: V, X, φ, ω.

Solution: Compute (see equations (40) and (43)),

$$T' u = \frac{T v \cos \omega}{C} = 2 z \left(\frac{e^{\frac{1}{2} n} - 1}{n e^{\frac{1}{2} n}} \right)$$

assuming for a first approximation

$$\cos \varphi = \cos \omega = 1$$

Find z from the table by inspection and trial and take out u, A, log B'. We then have approximately

$$V = \frac{800 v}{u}$$

$$X = C z$$

$$\sin 2 \varphi = A C \left(\frac{800}{V} \right)^2$$

$$\tan \omega = B' \tan \varphi$$

Repeat as often as may be necessary supplying the factors $\cos \varphi$ and $\cos \omega$ in the expression for V, and $\cos \omega$ in the expression for $T' u$.

Example. $\log C = 9.52575 - 10$, $v = 550$ f. s., $T = 4$ sec.

Answers. $V = 792$ f. s., $X = 2615$ ft.. $\varphi = 5°$, $\omega = 6° 21' 30''$.

Solution for metric units.

Compute C as in Problem One and $T' u$ by the equation

$$T' u = [0.51599] \frac{T v \cos \omega}{C}$$

and proceed as in English units.

PROBLEM THIRTY-FIVE.

Given: X, v, T. Required: C, V, φ, ω.

Solution: Compute (see equations (13), (40) and (43)),

$$\frac{T' u}{z} = \frac{T v \cos \omega}{X} = 2 \left(\frac{e^{\frac{1}{2} n} - 1}{n e^{\frac{1}{2} n}} \right)$$

and proceed as in Problem 34.

Example. $X = 1200$ yds., $v = 525$ f. s., $T = 6$ sec.

We have, by two approximations, $\log C = 9.73863$, $V = 710$ f.s., $\varphi = 8° 16'$, $\omega = 10° 6'$.

Solution for metric units.

Same as for English units.

We have now provided for all the cases that can arise employing only what we have called primary elements, in connection with horizontal trajectories, that is, when the target and gun are at the same level. But this would rarely be the case in field service except on our western plains or on the Atlantic and Gulf seaboard. In campaigns in rolling and especially in mountainous regions the guns and objects to be fired at will generally be on different levels, and it therefore becomes necessary to establish convenient formulas for these conditions, which we now proceed to do.

From equation (1) we have

$$S(u) = \frac{X}{C} + S(V)$$

and therefore the *pseudo* velocity at any point of the trajectory is independent of the height of this point above (or below) the level of the gun, but depends only, for the same projectile, on the muzzle velocity and the horizontal distance from the muzzle of the gun, which is the origin of coordinates. That is u is independent of y and depends only upon V and x. Therefore, making

$$\frac{A(u) - A(V)}{S(u) - S(V)} - I(V) = a$$

$$I(u) - \frac{A(u) - A(V)}{S(u) - S(V)} = b$$

$$I(u) - I(V) = a + b = a'$$

equations (2) and (3) become for any velocity V less than 800 f. s.,

$$\frac{y}{x} = \tan\varphi - \frac{a\,C}{2\cos^2\varphi}\left(\frac{800}{V}\right)^2 \tag{47}$$

$$\tan\theta = \tan\varphi - \frac{a'\,C}{2\cos^2\varphi}\left(\frac{800}{V}\right)^2 \tag{48}$$

But we have (equation (41)), for a velocity V,

$$\sin 2\varphi = A\,C\left(\frac{800}{V}\right)^2$$

Whence by substitution and reduction,

$$\frac{y}{x} = \frac{\tan\varphi}{A}\Big(A - a\Big) \tag{49}$$

$$\tan\theta = \frac{\tan\varphi}{A}\Big(A - a'\Big) \tag{50}$$

In these equations a and a' are functions of x/C, where x is the abscissa of any point of the trajectory; while A is a function of X/C, X being the entire horizontal range. That is, for the same trajectory A is constant while a and a' vary with x. It is evident that A and a are to be taken from the same column in Table 1 but with different arguments; and a' from the "A'" column. These equations, it will be observed, are independent of the muzzle velocity.

We may express y/x in another form which is sometimes useful as follows:—As has been said already a refers to the point of the trajectory x, y; but as the pseudo velocity is independent of the height of this point and depends only on x, or the horizontal distance from the muzzle of the gun, we may consider x as the range of another horizontal trajectory whose angle of departure may be designated by φ_x, and we may have

$$a = \frac{\sin 2\varphi_x}{C}\left(\frac{V}{800}\right)^2$$

as well as

$$A = \frac{\sin 2\varphi}{C}\left(\frac{V}{800}\right)^2$$

Substituting these in equation (49) we have by a slight reduction

$$\frac{y}{x} = \frac{1}{2\cos^2\varphi}\left\{\sin 2\varphi - \sin 2\varphi_x\right\} \tag{51}$$

This equation can also be changed to the form

$$\frac{y}{x} = \tan(\varphi - \varphi_x)\left\{1 - \frac{\sin^2\varphi_x}{\cos^2\varphi}\right\} \tag{52}$$

If, in equation (49), we make

$$\frac{y}{x} = \tan\varepsilon$$

ε will be the angular distance of the point x, y above (or below) the level of the gun. Substituting we have

$$\tan\varepsilon = \frac{\tan\varphi}{A}\left(A - a\right) \tag{53}$$

Subtracting this last from equation (50) gives

$$\tan\varepsilon - \tan\theta = \frac{\tan\varphi}{A}\left(a' - a\right)$$

But by defination, if ε and θ refer to the same point of the trajectory,

$$a' - a = b$$

Therefore we have this relation between the three angles φ, θ and ε, namely,

$$\tan\varepsilon - \tan\theta = \frac{b}{A}\tan\varphi$$

where b is to be taken from the "B" column of Table 1 with the argument x/C. Substituting for A its value from equation (41) and for b its value from equation (42). namely,

$$\tan \omega_x = \frac{b\,C}{2\cos^2 \varphi_x}\left(\frac{800}{V}\right)^2$$

we have finally the simple relation

$$\tan \varepsilon - \tan \theta = \tan \omega_x \left(\frac{\cos \varphi_x}{\cos \varphi}\right)^2 \qquad (54)$$

At the summit of a trajectory θ is zero, and for this point equation (50) reduces to the identity

$$a' = A$$

Substituting this in equation (49) and distinguishing symbols referring to the summit by zero subscripts we have

$$\frac{y_0}{x_0} = \frac{a'_0 - a_0}{a'_0} \tan \varphi = \frac{b_0}{a'_0} \tan \varphi$$

But we have

$$x_0 = C z_0$$

and therefore

$$y_0 = C \frac{b_0 z_0}{a'_0} \tan \varphi$$

Making

$$\frac{b_0 z_0}{a'_0} = A''$$

we have finally

$$y_0 = A''\, C \tan \varphi$$

which is equation (35) and is independent of the muzzle velocity.

We will now apply equations (47)–(54) to the solution of some important problems.

PROBLEM THIRTY-SIX.

Given: Any three of the primary elements of a horizontal trajectory. Required: The coordinates of the summit (x_0, y_0), and the summit velocity.

Solution: If C, V and φ are not known they must first be computed from the given data by means of the proper Problems. Then either take A from Table 1 or compute it by the formula

$$A = \frac{\sin 2\varphi}{C}\left(\frac{V}{800}\right)^2$$

Then, since at the summit $A' = A$, find the value of A in the "A′" column of Table 1 and take out the corresponding values of z, u, A'', designating the first two, since they refer to the summit, by z_0, u_0. We then have

$$x_0 = Cz_0$$
$$y_0 = A'' \, C \tan \varphi$$
$$v_0 = \frac{V}{800} u_0 \cos \varphi$$

Example 1. Compute x_0, y_0, v_0 for the trajectory of Ex. 2, Problem Twelve.

Here $\log C = 0.00816$, $V = 649.01$ f. s., $\varphi = 16° \, 51' \, 30''$ and $A = .35859$. Making

$$A' = 0.35859$$

we find by interpolation from Table 1,

$$z_0 = 3000 + \frac{100 \times 104.9}{133} = 3079$$
$$u_0 = 695.3 - .79 \times 3.3 = 692.7$$
$$A'' = 1570 + .79 \times 55 = 1613$$

Therefore by the formulas, $x_0 = 3137$ ft., $y_0 = 498$ ft., $v_0 = 537.8$ f. s.

Example 2. Compute the summit elements for the trajectory of Ex. 2, Problem Fifteen.

Here $\log C = 0.40333$, $V = 663.1$ f. s., $\varphi = 13° \, 6'$ and $A = A' = .11980$. Therefore from Table 1

$$z_0 = 1100 + \frac{100 \times 34}{112} = 1130,$$
$$u_0 = 759.9 - .3 \times 3.5 = 758.8,$$
$$A'' = 560 + .3 \times 51 = 575,$$

and by the formulas $x_0 = 2860$ ft., $y_0 = 338.7$ ft., $v_0 = 612.6$ f. s.

Solution for metric units.

Proceed as explained for English units, computing A (if not taken from Table 1) by the equation

$$A = [5.22580 - 10] \frac{V^2 \sin 2\varphi}{C}$$

which is also the summit value of A'. With this value of A' take z_0, u_0 and A'' from Table 1, and then compute x_0, y_0, v_0 by the formulas

$$x_0 = [9.48401 - 10] \, Cz_0$$
$$y_0 = [9.48401 - 10] \, A'' \, C \tan \varphi$$
$$v_0 = [7.09691 - 10] \, V u_0 \cos \varphi$$

Example 3. Compute the summit elements for the trajectory of Ex. 3, Problem Two.

Here log $C = 0.52562$, $V = 200$ m. s., $\varphi = 30°$, $A = .17369 = A'$. Therefore

$$z_0 = 1600 + \frac{100 \times 1.9}{117} = 1601.6$$

$$u_0 = 742.4 - \frac{1.9 \times 3.5}{117} = 742.3,$$

$$A'' = 820 + \frac{1.9 \times 52}{117} = 820.84,$$

and by the formulas $x_0 = 1637$ m, $y_0 = 484.5$ m, $v_0 = 160.7$ m. s.

Example 4. Compute the summit elements for the trajectory of Ex. 2, Problem Five.

Here log $C = 0.36276$, $V = 200$ m. s., $\varphi = 22° 6'$ and $A = .24619 = A'$. Therefore

$$z_0 = 2200 + \frac{5.9 \times 100}{123} = 2204.8,$$

$$u_0 = 721.8 - \frac{5.9 \times 3.4}{123} = 721.6,$$

$$A'' = 1138 + \frac{5.9 \times 53}{123} = 1140.5,$$

and by the formulas $x_0 = 1552$ m, $y_0 = 325.4$ m, $v_0 = 183.9$ m. s.

The maximum ordinate is of no special interest in itself but is useful for correcting the ballistic coefficient when it is considered necessary to take into account the mean altitude of the projectile.* The logarithm of the atmospheric factor f which is to be added to the uncorrected log C is found (for English units) by the equation

$$\log (\log f) = \log y'_0 + 5.01765 - 10$$

and for metric units

$$\log (\log f) = \log y'_0 + 5.53364 - 10$$

It is assumed in these formulas that the mean altitude of the projectile during its flight is two-thirds of the uncorrected maximum ordinate.

Example 5. Compute the range (corrected for the altitude) for the 7 inch siege mortar, muzzle velocity 690 f. s., angle of departure 45°.

Here $d = 7$ inches, $w = 125$ lbs., $c = 1$, $\beta = 1.17$, $V = 690$ f. s., $\varphi = 45°$.

The work may be arranged as follows :

---

* See *Handbook*, Problem XIII.

$$\begin{array}{rcl}
\log w & = & 2.09691 \\
\log (\beta d^2) & = & 1.75839 \\
\hline
\log (w/\beta d^2) & = & 0.33852 \\
\hline
\log V & = & 2.83885 \\
\log 800 & = & 2.90309 \\
\hline
 & & 9.93576 \\
 & & 2 \\
\hline
 & & 9.87152 \\
\log (w/\beta d^2) & = & 0.33852 \\
\hline
\log A' & = & 9.53300 \\
A' & = & 0.34120 \\
\end{array}$$

$$\therefore A'' = 1515 + \frac{63 \times 55}{132} = 1541$$

$$\begin{array}{rcl}
\log A'' & = & 3.18780 \\
\log (w/\beta d^2) & = & 0.33852 \\
\text{Const. } \log & = & 5.01765 \\
\hline
\log 0.03499 & = & 8.54397 \\
\log (w/\beta d^2) & = & 0.33852 \\
\hline
\log C & = & 0.37351 \\
\end{array}$$

$$\begin{array}{rcl}
2\,(\log V/800) & = & 9.87152 \\
\log C & = & 0.37351 \\
\hline
\log A & = & 9.49801 \\
A & = & 0.31478 \\
\end{array}$$

$$\therefore z = 5200 + \frac{54300}{702} = 5277.3$$

$$\begin{array}{rcl}
\log z & = & 3.72241 \\
\log C & = & 0.37351 \\
\hline
\log X & = & 4.09592 \\
X & = & 12472 \text{ ft.} = 4157 \text{ yds.} \\
\end{array}$$

Example 6. Compute the range of Ex. 4, page 13, corrected for altitude.

Answer. $\log C = 0.38687$, $X = 3628$ m.

PROBLEM THIRTY-SEVEN.

Given: y_0, C, φ. Required: The remaining elements of the trajectory.

Solution: Compute (see equation (35))

$$A'' = \frac{y_0}{C \tan \varphi}$$

with which take A' from the table and find its value in the A column. With this latter take out z, u, $\log B'$, T', and compute the elements as in Problems Three and Four.

Example 1. $C = 1$, $\varphi = 10°$, $y_0 = 50$ ft.

We find $A'' = 283.56$, $A' = A = .05796$, $z = 1113.7$. Therefore $X = 1113.7$ ft., $V = 329.3$ f. s.

As the muzzle velocity is quite small in this example, it will be interesting to see how the above range compares with that given by the parabolic theory. By this theory we have

$$X = 4\,y_0 \cot \varphi$$

Giving to y_0 and φ the values in the above example we find $X = 1134.3$ ft.

Solution for metric units.

Compute C as in Problem One and then A'' by the formula

$$A'' = [0.51599] \frac{y_0}{C \tan \varphi}$$

with which proceed as for English units, and in Problems Three and Four.

Example 2. $d = 24$ cm, $w = 144$ kg, $y_0 = 10$ m, $\varphi = 10°$.

The following is the work in full. For log C see Problem One.

$$\begin{aligned} \text{Const. log} &= 0.51599 \\ \log y_0 &= 1.00000 \\ \text{a. c. } \log C &= 9.47438 \\ \log \cot \varphi &= 0.75368 \\ \hline \log A'' &= 1.74405 \\ A'' &= 554.7 \end{aligned}$$

$$\therefore A' = A = .1053 + \frac{46.7 \times 111}{52} = .10629$$

$$\therefore z = 1900 + \frac{49000}{568} = 1986.3$$

$$\begin{aligned} \text{Const. log} &= 9.48401 \\ \log C &= 0.52562 \\ \log z &= 3.29805 \\ \hline \log X &= 3.30768 \\ X &= 2031 \text{ m.} \end{aligned}$$

$$\begin{aligned} \log A &= 9.02649 \\ \log C &= 0.52562 \\ \log \operatorname{cosec} 2\varphi &= 0.46595 \\ \hline 2)\ &\ 0.01806 \\ \hline &\ 0.00903 \\ \text{Const. log} &= 2.38710 \\ \hline \log V &= 2.39613 \\ V &= 249 \text{ m.s.} \end{aligned}$$

PROBLEM THIRTY-EIGHT.

Given: The elements of a trajectory. Required: The equation of the trajectory and the value of y for any given value of x.

Solution: Solved by equation (49), namely,

$$y = \frac{\tan \varphi}{A} (A - a)\, x$$

In this equation, it must be remembered, A refers to the complete horizontal trajectory and is either taken from Table 1 with the argument

$$z = \frac{X}{C}$$

or computed by the formula

$$A = \frac{\sin 2\varphi}{C} \left(\frac{V}{800}\right)^2$$

according to the given data. The quantity a varies with the abscissa x and is taken from the same column as A with the argument

$$z = \frac{x}{C}$$

Approximate Solution for flat trajectories.

In equation (52) when φ and φ_x are small, the term

$$\frac{\sin^2 \varphi_x}{\cos^2 \varphi}$$

may be omitted in comparison with unity and the equation reduces to the very simple expression

$$y = x \tan (\varphi - \varphi_x) \tag{55}$$

If we have a table of fire for the gun in question φ_x can be taken directly from it with the range x. If not φ_x must be computed by the formula

$$\sin 2\,\varphi_x = a\,C\left(\frac{800}{V}\right)^2$$

Example 1. $C = 2$, $X = 4000$ ft., $\varphi = 10°$, $x = 3000$ ft.

With the two arguments $z = 2000$ and $z = 1500$ we find from Table 1, $A = .10707$ and $a = .07902$. The equation of this trajectory is therefore

$$y = [0.21665]\ (.10707 - a)\ x$$

Making $a = .07902$ and $x = 3000$ gives $y = 138.58$ ft. We also have by Problem Three $V = 633$ f. s.

Working this example by equation (55) we find

$$\varphi_x = 7°\ 18'\ 40'' \text{ and } y = 140.9 \text{ ft.}$$

The difference between these two values of y of course disappears if we restore the term

$$\left(\frac{\sin \varphi_x}{\cos \varphi}\right)^2$$

to equation (52). The labor is about the same by both methods so that there is nothing gained by employing equation (55) unless we have a table of fire from which to take the angle φ_x.

Example 2. $C = 1$, $V = 700$ f. s., $\varphi = 20°$, $x = 3000$ ft.

In this example a must be taken from Table 1 with the argument $z = 3000$, while A must be computed by the formula

$$A = \frac{\sin 2\,\varphi}{C}\left(\frac{V}{800}\right)^2$$

We find $A = .49213$ and $a = .16590$. Whence the equation of the trajectory is

$$y = [9.86899 - 10]\ (.49213 - a)\ x$$

When $x = 3000$ ft., $y = 723.8$ ft.

Example 3. $C = 1$, $V = 700$ f. s., $X = 3000$ ft., $x = 100$ ft.

In this example take A and a from Table 1 with the arguments $z = 3000$ and $z = 100$. Then compute φ by Problem One and y as already illustrated.

Answers.

$$\varphi = 6^\circ\ 15'\ 25''$$
$$y = [9.82012 - 10]\ (.16590 - a)\ x$$
When $x = 100$ ft., $y = 10.63$ ft.

By equation (55) $y = 10.57$ ft.

Example 4. Suppose that in Ex. 2 of Problem Fifteen, there was a ridge between the position selected for the gun and the escarp to be breached, whose crest was 80 ft. above the level of the gun and 100 yards from the escarp. Would the shot clear the crest?

Here from Problem Fifteen and the new data, $X = 1875$ yards, $\varphi = 13^\circ\ 6'$, $z = 2221.7$, $A = 0.11979$, $x = 5325$ ft. and log C 0.40333, to find y. The new value of z from which to get a from the table is $5325/C = 2103.7$, from which $a = .11300$. We next find log ($\tan\varphi/A$) to be 0.29295. The general expression for y for this trajectory is therefore

$$y = [0.29295] \left\{ 0.11979 - a \right\} x$$

and supplying the values of a and x for this particular case, we find

$$y = 70.98 \text{ ft.}$$

The projectile would then strike the ridge 9 ft. below its crest. To carry the projectile over the ridge and not diminish the striking velocity the gun must be placed further from the escarp, assuming the ground to be level. Suppose the gun to be placed 300 yards further from the escarp, would the projectile then clear the ridge? In this case we have $X = 2175$ yds. $= 6525$ ft., log $C = 0.40313$ and $v = 600$ f.s. to find the other elements of the trajectory by Problem Twenty-one, and y as above. In computing V by Problem Twenty-one we will employ the values of φ and ω already given or found, since although they will both be increased by the new data, still the *ratio* of their cosines will be sensibly unaltered. We find $z = 2577.8$. $u = 709.1$, $A = .14061$, log $B' = 0.03488$. Whence $V = 674.25$ f. s., $\varphi = 15^\circ\ 2'\ 5''$, $\omega = 16^\circ\ 14'$. To compute y we have $x = 6225$ ft., $x/C = 2459.3$ and $a = .13363$, by means of which we find $y = 83.00$ ft. The projectile would then clear the ridge by three feet. We may now determine the distance of the gun from the escarp so that the projectile would just graze the ridge as follows: For a range of 5625 ft. the trajectory passes 9.02 ft. below the crest of the ridge; and for a range of 6525 ft. the trajectory is too high by 3 ft. We therefore have the proportion

$$12:900::3:225$$

whence the required range is $6525 - 225 = 6300$ ft. With this value of X we find $z = 2488.9$, and from Table 1, $u = 712.2$, $A = .13537$, $\log B' = 0.03368$. Whence $V = 670$ f. s., $\varphi = 14° 37' 15''$, $\omega = 15° 44' 30''$. The value of y computed for these last data is 80.24 ft. The problem is therefore completely solved.

Solution for metric units.

Compute C and z as in Problem One and then y as for English units.

Example 5. Compute the ordinate for the trajectory of Ex. 4, Problem One, when $x = 400$ m.

Here $\log C = 0.52562$, $A = .17681$, $\varphi = 30° 55'$. The equation to the trajectory is easily found to be

$$y = [0.52983]\,(0.17681 - a)\,x$$

We have first to find from the A column of Table 1 the value of a for the given value of x, with the argument z computed by the equation

$$z = [0.51599]\,\frac{x}{C}$$

We find $z = 391.23$, $a = 0.01990$ and $y = 212.4$ m.

PROBLEM THIRTY-NINE.

Given: The elements of a horizontal trajectory. Required: The value of x for any given value of y.

Solution: Substituting Cz for x in equation (49) and transposing, we have

$$(A - a)\,z = \frac{Ay}{C}\cot\varphi$$

The second member of this equation consists of known quantities while in the first member a and z are both unknown. They can easily be found however from Table 1 by inspection and trial. We then have

$$x = Cz$$

Example 1. $C = 1$, $V = 800$ f. s., $\varphi = 5°$, $y = 50$ ft.

By Problem Two we find $A = .17365$, and therefore the trial equation becomes

$$\log\,(.17367 - a)\,z = 1.99669$$

This equation is satisfied when $z = 728.7$ and also 2451; and since the ballistic coefficient in this example is unity these are the two values of x required, in feet.

The corresponding abscissas *in vacuo* are 722.4 and 2732.4 feet.

Solution for metric units.

The trial equation for metric units is

$$(A - a) z = [0.51599] \frac{A y}{C \tan \varphi}$$

Having determined z from Table 1 by trial compute the required abscissa by the equation

$$x = [9.48401 - 10] \, C z$$

Example 2. With the data of Ex. 4, Problem One, compute the longer abscissa corresponding to $y = 1\frac{1}{2}$ m.

Employing numbers already found in the example referred to, the operations are as follows:

$$\begin{aligned} \log A &= 9.24751 \\ \log y &= 0.17609 \\ \text{a. c. } \log C &= 9.47438 \\ \log \cot \varphi &= 0.22266 \\ \text{Const. } \log &= 0.51599 \\ \hline \log 0.43314 &= 9.63663 \end{aligned}$$

The trial equation is therefore

$$(.17681 - a) z = 0.43314$$

which is found to be satisfied when

$$\begin{aligned} z &= 5176.4 \\ \log z &= 3.50194 \\ \log C &= 0.52562 \\ \text{Const. } \log &= 9.48401 \\ \hline \log x &= 3.51157 \\ x &= 3247.7 \text{ m.} \end{aligned}$$

The only practical application of this problem relates to that part of the trajectory near the striking point, in determining the breadth of the danger zone; and this is of less importance in curved than in direct fire. In the last example the breadth of the danger zone would be

$$X - x = 2.3 \text{ m.}$$

In all operations where curved fire is employed we may determine the breadth of the danger zone with sufficient accuracy by the formula

$$z = y \cot \omega$$

PROBLEM FORTY.

Given: The elements of a horizontal trajectory. Required: The value of θ for any given value of x.

Solution: Solved by equation (50), namely,

$$\tan \theta = \frac{\tan \varphi}{A} (A - a')$$

In this equation A is either taken from Table 1 with the argument X/C; or, if X is not known, A may be computed by the equation

$$A = \frac{\sin 2\varphi}{C} \left(\frac{V}{800}\right)^2$$

a' is to be taken from the "A′" column with the argument x/C.

Approximate Solution for flat trajectories.

We have (see equation (54))

$$\tan \theta = \tan \varepsilon - \tan \omega_x \left(\frac{\cos \varphi_x}{\cos \varphi}\right)^2$$

Or, if these angles are all small, we may write, approximately,

$$\theta = \varepsilon - \omega_x$$

The angle ω_x can be taken directly from a table of fire; or it can be computed by either equation (18) or (42) according to the known data.

This approximate method requires that y be first computed by Problem Thirty-eight in order to get ε. It has no advantage over the complete method.

Example 1. Deduce the general expression for θ for the trajectory in which $C = 2$, $X = 4000$ ft. and $\varphi = 10°$. Also compute θ when $x = 3000$ ft. (see Ex. 1, page 45).

Answers.

$$\tan \theta = [0.21665] (.10707 - a')$$
$$\theta = -5° 9'$$

Example 2. Data of Ex. 2, page 45.

Answers.

$$\tan \theta = [9.86899 - 10] (.49213 - a')$$
$$\theta = 6° 5'$$

Example 3. Data of Ex. 3, page 46.

Answers.

$$\tan \theta = [9.82012 - 10] (.16590 - a')$$
$$\theta = 5° 52' 50''$$

It will be observed that the numerical parts of the general expression for tan θ are the same as those for y; and that the sign of

θ depends upon whether a' is greater or less than A. When these are equal θ becomes zero and the point under consideration is the summit of the trajectory. Finally it may be remarked that the labor of computing θ is not lessened by using the approximate method unless we can take ω_x from a table of fire;—and then but very little.

Solution for metric units.

Compute C, z and A by the formulas of Problems One and Two, and then proceed as for English units. The muzzle velocity can be computed by Problem Three.

Example 4. Deduce the general expression for θ for the trajectory in which $\log C = 0.52562$, $X = 2000$ m, and $\varphi = 10°$. Also compute θ when $x = 1500$ m. (see Ex. 4, page 11).

Solution:

$$\begin{aligned} \text{Const. log} &= 0.51599 \\ \log X &= 3.30103 \\ \text{a. c. log } C &= 9.47438 \\ \hline \log z &= 3.29140 \\ z &= 1956.15 \end{aligned}$$

$$\therefore A = .10139 + .5615 \times .00568 = .10458$$

$$\begin{aligned} \log \tan \varphi &= 9.24632 \\ \log A &= 9.01945 \\ \hline &\ 0.22687 \end{aligned}$$

$$\therefore \tan \theta = [0.22687]\,(.10458 - a')$$

$$\begin{aligned} \text{Const. log} &= 0.51599 \\ \log x &= 3.17609 \\ \text{a. c. log } C &= 9.47438 \\ \hline \log z &= 3.16646 \\ z &= 1467.1 \end{aligned}$$

$$\therefore a' = .1503 + .671 \times .0115 = .1580$$

$$\begin{aligned} A &= .10458 \\ a' &= .15800 \\ \hline \log - .05342 &= 8.72770_n \\ \text{Const. log} &= 0.22687 \\ \hline \log \tan \theta &= 8.95457_n \\ \theta &= -\ 5°\ 9' \end{aligned}$$

We also find by Problem Three, $V = 247$ m. s.

PROBLEM FORTY-ONE.

Given: The ballistic coefficient, muzzle velocity and rectangular coordinates of any point of a horizontal trajectory. Required: The remaining elements of the trajectory.

Solution: Making

$$\tan \varepsilon = \frac{y}{x}$$

equation (47) becomes

$$\tan \varepsilon = \tan \varphi - \frac{a\,C}{2 \cos^2 \varphi} \left(\frac{800}{V}\right)^2$$

If φ_x is the angle of departure for the horizontal range x and given muzzle velocity V, we have by Problem One

$$\sin 2\,\varphi_x = a\,C \left(\frac{800}{V}\right)^2$$

Therefore

$$\tan \varepsilon = \tan \varphi - \frac{\sin 2\,\varphi_x}{2 \cos^2 \varphi}$$

or

$$\sin 2\,\varphi - 2 \cos^2 \varphi \tan \varepsilon = \sin 2\,\varphi_x \tag{58}$$

But

$$2 \cos^2 \varphi = 1 + \cos 2\,\varphi$$

whence

$$\sin 2\,\varphi - \cos 2\,\varphi \tan \varepsilon - \tan \varepsilon = \sin 2\,\varphi_x$$

Multiplying and dividing the first member by $\cos \varepsilon$ we have

$$\frac{\sin 2\,\varphi \cos \varepsilon - \cos 2\,\varphi \sin \varepsilon - \sin \varepsilon}{\cos \varepsilon} = \sin 2\,\varphi_x$$

therefore

$$\frac{\sin (2\,\varphi - \varepsilon) - \sin \varepsilon}{\cos \varepsilon} = \sin 2\,\varphi_x$$

and finally

$$\sin (2\,\varphi - \varepsilon) = \sin \varepsilon\,(1 + \cot \varepsilon \sin 2\,\varphi_x) \tag{59}$$

In computing φ_x by equation (57) a is to be taken from the "A" column of Table 1 with the argument x/C. Having computed φ by equation (59) we then know C, V and φ by means of which the remaining elements of the trajectory passing through the point x, y can be computed by Problem One.

Example 1. $C = 2$, $V = 700$ f. s., $x = 3000$ ft., $y = 1000$ ft.

Here $\tan \varepsilon = \frac{1}{3}$, $\varepsilon = 18^\circ\ 26'\ 10''$, $z = 1500$ and $a = 0.07902$.

The computations may conveniently be arranged as follows:

$$\begin{aligned} \log a &= 8.89774 \\ \log C &= 0.30103 \\ 2 \log 800/V &= 0.11598 \\ \hline \log \sin 2\,\varphi_x &= 9.31475 \quad \therefore \varphi_x = 5^\circ\ 57'\ 25'' \end{aligned}$$

$$\begin{aligned}
\log \tan \varepsilon &= \underline{9.52288} \\
\log 0.61926 &= \underline{9.79187} \\
\log 1.61926 &= 0.20932 \\
\log \sin \varepsilon &= \underline{0.50000} \\
\log \sin (2\varphi - \varepsilon) &= 9.70932 \\
2\varphi - \varepsilon &= 30^\circ\ 48' \\
\varepsilon &= \underline{18\ \ 26} \\
2\varphi &= 49\ \ 14 \\
\varphi &= 24^\circ\ 37'
\end{aligned}$$

Example 2. Observations upon the enemy's captive balloon show that the coordinates of its position are $x = 4800$ ft. and $y = 1600$ ft. It is desired to open fire upon it with shell from a field mortar for which $C = 2$ and $V = 800$ f. s. Determine the angle of departure, striking angle, striking velocity and time of flight.

Here $\tan \varepsilon = \frac{1}{3}$, $\varepsilon = 18^\circ\ 26'\ 10''$, $z = 2400$ and $a = .13016$. We find $\varphi = 26^\circ\ 21'\ 25''$, $\theta = 9^\circ\ 8'$, $v = 649$ f. s., $t = 7.1$ sec. and $X = 4277$ yards. The balloon is therefore in the ascending branch of the trajectory, and will be struck from below.

Approximate Solution for flat trajectories.

If the angle of departure φ is small, equation (58) may be written, approximately, (since ε and φ_x are both less than φ),

$$\varphi = \varepsilon + \varphi_x \tag{60}$$

Example 3. Compute φ by equation (60) with the data of Ex. 1. We have

$$\begin{aligned}
\varepsilon &= 18^\circ\ 26' \\
\varphi_x &= \underline{\ \ 5\ \ 57} \\
\varphi &= 24^\circ\ 23'
\end{aligned}$$

which is 14′ too small. A nearer approximation is obtained by taking instead of x the distance from the gun to the target. Calling this distance s we have

$$s = \sqrt{x^2 + y^2} = x \sec \varepsilon.$$

In Ex. 3 we have $s = 3162.3$, $z = 1581.15$ and $a = .08352$. Therefore $\varphi = 24^\circ\ 44'$ which is to great by 7′.

This is an extreme case. If we make $y = 100$ ft., the other data remaining the same, we shall find by equation (60)

$$\varphi = 7^\circ\ 52'$$

which is but one minute of arc less than that obtained by the complete equation (59).

Solution for metric units.

Compute sin 2 φ_x as explained in Problem Two for metric units and then proceed as for English units.

PROBLEM FORTY-TWO.

Given: The muzzle velocity, angle of projection and rectangular coordinates of the target. Required: The remaining elements of the trajectory.

Solution: From equation (58) we have

$$\begin{aligned}\sin 2\varphi_x &= \sin 2\varphi - 2\cos^2\varphi \tan\varepsilon \\ &= \sin 2\varphi\,(1 - \cot\varphi \tan\varepsilon)\end{aligned}$$

by means of which φ_x can be computed. Then knowing V, x and φ_x we can compute C by Problem Five and then the remaining elements of the trajectory by Problem Two.

Example 1. Firing with a certain gun which gave to the projectile a muzzle velocity of 790 f. s., and with an angle of projection of 10°, the projectile was observed to strike on the side of a bluff 150 feet above the gun and distant 5000 feet (horizontal). What was the ballistic coefficient?

Here $V = 790$ f. s., $\varphi = 10°$, $x = 5000$ ft., $y = 150$ ft. and $\tan\varepsilon = 0.03$.

The work may be conveniently arranged as follows:

$$\begin{aligned}\log\tan\varepsilon &= 8.47712 \\ \log\tan\varphi &= 9.24632 \\ \hline \log 0.17014 &= 9.23080 \\ \hline \log 0.82986 &= 9.91900 \\ \log\sin 2\varphi &= 9.53405 \\ \hline \log\sin 2\varphi_x &= 9.45305 \\ 2\log V/800 &= 9.98908 \\ \hline & \ 9.44213 \\ \log x &= 3.69897 \\ \hline \log C' &= 5.74316\end{aligned}$$

$$\therefore z = 3000 + \frac{460}{14} = 3033$$

$$\begin{aligned}\log V &= 2.89763 \\ \log 800 &= 2.90309 \\ \hline & \ 9.99454 \\ \hline \log x &= 3.69897 \\ \log z &= 3.48187 \\ \hline \log C &= 0.21710\end{aligned}$$

Solution for metric units.

Compute log sin 2 φ_x as explained above for English units, and then log C by Problem Five, and the remaining elements by Problem Two.

Example 2.* $V = 202$ m. s., $\varphi = 27°$, $x = 3100$ m and $y = 42$ m. Compute log C, X, ω, v and T.

We give the work in full.

$$\begin{aligned} \log y &= 1.62325 \\ \log x &= 3.49136 \\ \hline \log \tan \varepsilon &= 8.13189 \\ \log \tan \varphi &= 9.70717 \\ \hline \log 0.02659 &= 8.42472 \\ \hline \log 0.97341 &= 9.98830 \\ \log \sin 2\varphi &= 9.90796 \\ \hline \log \sin 2\varphi_x &= 9.89626 \\ 2 \log V &= 4.61070 \\ \text{Const. } \log &= 4.70981 \\ \hline & \ 9.21677 \\ \log x &= 3.49136 \\ \hline \log C' &= 5.72541 \end{aligned}$$

$$\therefore z = 1700 + \frac{910}{13} = 1770$$

$$\begin{aligned} \text{Const. } \log &= 0.51599 \\ \log x &= 3.49136 \\ \text{a. c. } \log z &= 6.75203 \\ \hline \log C &= 0.75938 \\ \log \tan \varphi &= 9.70717 \\ \log B' &= 0.02457 \\ \hline \log \tan \omega &= 9.73174 \\ \omega &= 28° 20' \end{aligned}$$

(Prob. 2)

$$\begin{aligned} \text{Const. } \log &= 5.22580 \\ 2 \log V &= 4.61070 \\ \log \sin 2\varphi &= 9.90796 \\ \text{a. c. } \log C &= 9.24062 \\ \hline \log A &= 8.98508 \\ A &= 0.09662 \end{aligned}$$

$$\therefore z = 1800 + \frac{8800}{565} = 1815.57$$

$$\begin{aligned} \log B' &= 0.02457 \\ T' &= 2.369 \\ u &= 735 \\ \text{Const. } \log &= 9.48401 \\ \log z &= 3.25901 \\ \log C &= 0.75938 \\ \hline \log X &= 3.50240 \\ X &= 3180 \text{ m.} \\ \text{Const. } \log &= 7.09691 \\ \log V &= 2.30535 \\ \log u &= 2.86629 \\ \log \cos \varphi &= 9.94988 \\ \log \sec \omega &= 0.05542 \\ \hline \log v &= 2.27385 \\ v &= 187.9 \text{ m. s.} \\ \text{Const. } \log &= 2.38710 \\ \log C &= 0.75938 \\ \log T' &= 0.37457 \\ \text{a. c. } \log V &= 7.69465 \\ \log \sec \varphi &= 0.05012 \\ \hline \log T &= 1.26582 \\ T &= 18.44 \text{ sec.} \end{aligned}$$

PROBLEM FORTY-THREE.

Given: The ballistic coefficient, angle of departure and rectangular coordinates of the target. Required: The remaining elements of the trajectory.

---

* Vallier, *Balistique Expérimentale*, page 109.

Solution: Compute $\sin 2\,\varphi_x$ as in Problem Forty-two. Then with C, x and φ_x compute V and the remaining elements by Problem Three, changing A to a and $\sin 2\,\varphi$ to $\sin 2\,\varphi_x$.

Example 1. Firing with a howitzer for which $C = 2$, and with an angle of departure of 15° the projectile was observed to strike on the side of a bluff 200 feet above the gun and distant 2400 yds. (horizontal). What was the muzzle velocity?

Here $C = 2$, $\varphi = 15°$, $x = 7200$ ft., $y = 200$ ft., $\tan \varepsilon = 1/36$, $z = 3600$ and $a = 0.20304$.

$$\begin{array}{rll}
\log \tan \varepsilon = & 8.44370 & \\
\log \tan \varphi = & \underline{9.42805} & \\
\log 0.10367 = & \underline{9.01565} & \\
\log 0.89633 = & 9.95247 & \\
\log \sin 2\,\varphi = & \underline{9.69897} & \\
\log \sin 2\,\varphi_x = & 9.65144 & \varphi_x = 13° \; 19' \\
\log a\,C = & \underline{9.60861} & \\
2\,) & \underline{9.95717} & \\
& 9.97858 & \\
\log 800 = & \underline{2.90309} & \\
\log V = & 2.88167 & \\
V = & 761.5 \text{ f. s.} &
\end{array}$$

It will be observed that φ_x, or rather $\log \sin 2\,\varphi_x$, is used simply as an auxiliary quantity.

This is an easy and accurate solution of an important problem. There is no saving of labor by computing φ_x by the approximate equation (60).

Example 2. Referring to Ex. 2, page 52, suppose the mortar were given the customary elevation of 45°, what muzzle velocity would be required to hit the balloon?

Here $C = 2$, $\varphi = 45°$, $x = 4800$ ft., $y = 1600$ ft., $\tan \varepsilon = \frac{1}{3}$, $z = 2400$ and $a = .13016$.

We find $V = 499.9$ f. s. and $\theta = -21° \; 3'$. The balloon is in the descending branch of this trajectory, and will be struck from above.

Solution for metric units.

Proceed as in Problems Forty-two, Three and One.

Example 3. $C = 5$, $\varphi = 20°$, $x = 3500$ m, $y = 50$ m.

Solution:

$$
\begin{array}{rl}
\log y = & 1.69897 \\
\log x = & 3.54407 \\
\hline
\log \tan \varepsilon = & 8.15490 \\
\log \tan \varphi = & 9.56107 \\
\hline
0.03925 = & 8.59383 \\
\hline
0.96075 = & 9.98261 \\
\log \sin 2\varphi = & 9.80807 \\
\hline
\log \sin 2\varphi_x = & 9.79068 \\
\log a\,C = & 9.79285 \\
2\,) & 0.00217 \\
 & 0.0010\dot{8}\ \text{(Prob. 3.)} \\
\text{Const. } \log = & 2.38710 \\
\hline
\log V = & 2.38818 \\
V = & 244.4\ \text{m. s.}
\end{array}
$$

$$
\begin{array}{rl}
\text{Const. } \log = & 0.51599 \\
\log x = & 3.54407\ \text{(Prob. 1.)} \\
\text{a. c. } \log C = & 9.30103 \\
\hline
\log z = & 3.36109 \\
z = & 2296.6 \\
\therefore\ a = & .11854 + .966 \times .00579 \\
= & .12413 \\
a\,C = & .62065
\end{array}
$$

PROBLEM FORTY-FOUR.

Given: The ballistic coefficient, the rectangular coordinates of a target above (or below) the level of the gun and the striking velocity. Required: The remaining elements of the trajectory.

Solution:

Equation (5) can be written for any muzzle velocity V less than 800 f. s.,

$$u = v \sec \varphi \cos \theta \left(\frac{800}{V}\right)$$

where u, v and θ refer to the point of the trajectory x, y, that is, to the target. From this we find

$$V = v \sec \varphi \cos \theta \left(\frac{800}{u}\right)$$

Of the factors comprised in the second member of this last equation we know v, and u can be taken from Table 1 with the argument x/C. We do not know the angles φ and θ, but for a first approximation we may assume

$$\sec \varphi \cos \theta = 1$$

and compute V by the approximate formula

$$V = \frac{800\,v}{u}$$

Next compute φ and θ by Problems Forty-one and Forty, and then V by the complete formula given above. It will generally be necessary to repeat this operation once more in order to get the correct value of V. Knowing C, V and φ the remaining elements can be computed by Problem Two. There is one point in the descending branch where the assumption made above with reference to the angles θ and φ is correct, namely, at the point where

$$\theta = -\varphi$$

To determine the coordinates of this point we have from equation (50) by making

$$\tan\theta = -\tan\varphi$$

the necessary condition

$$a' = 2A$$

Example. In Ex. 2, Problem Fifteen, suppose the place selected for the gun had been 1875 yards from the escarp to be breached and 100 feet below it. What muzzle velocity and angle of departure would have been necessary to secure a striking velocity of 600 f. s., and what would the striking angle have been?

Here $\log C = 0.40333$, $x = 5625$ ft., $y = 100$ ft., $v = 600$ f. s. and $\varepsilon = 1° \; 1' \; 7''$. We have

$$z = \frac{x}{C} = 2222.2$$

$$u = 721$$

$$V = \frac{800 \times 600}{721} = 666$$

With this value of V we find $\varphi = 14° \; 3'$ and $\theta = -13° \; 2'$. Making the proper corrections we next have $V = 668.6$ f. s, $A = .12909$, $a' = .24830$, $\varphi = 13° \; 57'$ and $\theta = -12° \; 55'$. These results need no further correction.

Example 2. Determine the coordinates of the point of the trajectory of Ex. 1 for which

$$\theta = -\varphi$$

Making

$$a' = 2A = .25818$$

we find from Table 1

$$z = 2300 + \frac{280}{125} = 2302.24$$

and

$$a = .12433 + .0224 \times .00583 = .12446$$

Substituting these values in equation (53) we have

$$\log\tan\varepsilon = 7.95307 - 10$$

We also have from equations (1) and (13)

$$x = Cz$$

and from equations (49) and (53)

$$y = x \tan \varepsilon$$

Therefore $x = 5827$ ft. and $y = 52.3$ ft.

Solution for metric units.

The expression for tan ε is independent of the units employed, and also V, if log C and z are computed by the formulas of Problem One.

PROBLEM FORTY-FIVE.

Given: C, x, y, θ. Required: The remaining elements of the trajectory.

Solution: By a proper combination of equations (47) and (48) we obtain the relation

$$\tan \varphi = \frac{1}{b'} (\tan \varepsilon - \tan \theta) + \tan \varepsilon \qquad (61)$$

and from equation (53)

$$A = \frac{a \tan \varphi}{\tan \varphi - \tan \varepsilon}$$

The remaining elements can now be computed by Problem Three.

The above expression for tan φ can be transformed to

$$\tan \varphi = (\tan \varepsilon - \tan \theta) \frac{\tan \omega_x}{\tan \varphi_x} + \tan \varepsilon$$

which, when the angles φ_x and ω_x do not differ greatly from each other, or in other words, when b' is near unity, may be written

$$\tan \varphi = 2 \tan \varepsilon - \tan \theta \qquad (62)$$

or, if all the angles and muzzle velocity are small, we may have

$$\varphi = 2\varepsilon - \theta \qquad (63)$$

In using equation (61) we must take log b' from the "log B" column of Table 1 with the argument $z = x/C$. The tangents of the angles can be taken from Table 4.

Example 1. $C = 2$, $x = 3000$ ft., $y = 300$ ft., $\theta = -10°$.

Solution: $z = \frac{3000}{2} = 1500$

$$\therefore a = 0.07902$$
$$\log b' = 0.02031$$

$$\begin{array}{rl}
\tan \varepsilon = & 0.10000 \\
\tan \theta = & -\ 0.17633 \\
\hline
\end{array}$$

$$\begin{array}{rl}
\log 0.27633 = & 9.44143 \\
\log b' = & 0.02031 \\
\hline
\log 0.26371 = & 9.42112 = \log(\tan \varphi - \tan \varepsilon) \\
\end{array}$$

$$\begin{array}{rl}
\tan \varepsilon = & 0.10000 \\
\hline
\tan \varphi = & 0.36371 \qquad \therefore\ \varphi = 19^\circ\ 59'
\end{array}$$

$$\begin{array}{rl}
\log a = & 8.89774 \\
\log \tan \varphi = & 9.56067 \\
\text{a. c. } \log(\tan \varphi - \tan \varepsilon) = & 0.57888 \\
\hline
\log A = & 9.03729 \\
\log C = & 0.30103 \\
\text{a. c. } \log \sin 2\varphi = & 0.19223 \\
\hline
2\,) & 9.53055 \\
\hline
 & 9.76527 \\
\log 800 = & 2.90309 \\
\hline
\log V = & 2.66836 \\
V = & 466 \text{ f. s.}
\end{array}$$

Therefore the projectile of this example would have to be fired with an angle of departure of 19° 59′ and a velocity of 466 f.s. in order to pass through the given point with the given inclination.

By equation (62) we have as follows:

$$\begin{array}{rl}
2 \tan \varepsilon = & 0.20000 \\
\tan \theta = & -\ 0.17633 \\
\hline
\tan \varphi = & 0.37633 \\
\varphi = & 20^\circ\ 37'
\end{array}$$

differing by 38′ from the complete solution. The error by using equation (63) would be still greater, namely, 1° 26′. It may be remarked that equation (62) is the solution of this problem by the parabolic theory, and therefore by employing it we throw out the resistance of the air.

The expression for the muzzle velocity in the parabolic theory for this problem is

$$V^2 = \frac{g\,x \cos \varepsilon}{2 \cos \varphi \sin(\varphi - \varepsilon)}$$

which gives for the data of Ex. 1, $V = 446.6$ f. s. Leaving out the resistance of the air then reduces muzzle velocity, which is what one would expect; but it also increases the angle of departure.

Example 2. In Ex. 2, Problem Fifteen suppose that with reference to the place selected for the gun the coordinates of the point to be breached are $x = 1875$ yds. and $y = 100$ ft. If the angle of descent must necessarily be 14°, what will be the muzzle velocity, angle of departure and striking velocity?

Here $\log C = 0.40333$, $x = 5625$ ft., and $y = 100$ ft.

Answers. $V = 645.9$ f. s., $\varphi = 14° 57'$ and $v = 575.9$ f. s.

Example 3. Referring to Ex. 2, suppose the gun to be 100 feet above the level of the point to be breached, the other data remaining the same.

Here $y = -100$ ft.

Answers. $V = 683.3$ f. s., $\varphi = 11° 13'$ and $v = 627.7$ f. s.

Solution for metric units.

Compute $\log C$ and z as in Problem One, φ as explained above for English units, A by Problem Two and V by Problem Three.

Example 4.* In preparing to breach a certain escarp suppose there are found two practicable positions for the guns, and we wish to determine which of the positions is best for the purpose. The data for the two positions as determined by the Engineers, are as follows, designating the two positions by A and B:

Position A.	*Position B.*
$x = 2485$ m	$x = 1040$ m
$y = 55.12$ m	$y = -27.88$ m
$\theta = -12° 16'$	$\theta = -12° 34'$
$\tan \varepsilon = 0.02218$	$\tan \varepsilon = -0.02681$

For the gun we have

$$d = 21 \text{ cm}$$
$$w = 78.75 \text{ kg}$$
$$c = 1.215$$
$$\text{whence, } \log C = 0.32022$$

We give the computations for the angles of projection and muzzle velocities in full.

---

* The data for this example are taken from Colonel La Llave's *Problemas de Balistica aplicados a la Fortificacion y a la Tactica.* Madrid, 1896, page 123, *et seq.*

Position A.

Const. log = 0.51599
log x = 3.39533
a. c. log C = 9.67978

log z = 3.59110
z = 3900.3
u = 666.6
a = .22217
log b' = 0.05275

tan ε = 0.02218
tan θ = − 0.21743

log 0.23961 = 9.37950
log b' = 0.05275

log 0.21220 = 9.32675
tan ε = 0.02218

tan φ = 0.23438
φ = 13° 11′.4

log a = 9.34669
log tan φ = 9.36989
a.c. log(tan φ − tan ε) = 0.67325

log A = 9.38983
log C = 0.32022
a. c. log sin 2 φ = 0.35230
2)0.06235
0.03117
Const. log = 2.38710

log V = 2.41827
V = 262.0 m. s.

log V = 2.41827
log u = 2.82387
log cos φ = 9.98839
log sec θ = 0.01003
a. c. log 800 = 7.09691

log v = 2.33747
v = 217.5 m. s.

Position B.

Const. log = 0.51599
log x = 3.01703
a. c. log C = 9.67978

log z = 3.21280
z = 1632.3
u = 741.3
a = .08636
log b' = 0.02210

tan ε = − 0.02681
tan θ = − 0.22292

log 0.19611 = 9.29250
log b' = 0.02210

log 0.18638 = 9.27040
tan ε = − 0.02681

tan φ = 0.15957
φ = 9° 4′

log a = 8.93631
log tan φ = 9.20297
a.c. log(tan φ − tan ε) = 0.72960

log A = 8.86888
log C = 0.32022
a. c. log sin 2 φ = 0.50692
2)9.69602
9.84801
Const. log = 2.38710

log V = 2.23511
V = 171.8 m. s.

log V = 2.23511
log u = 2.86999
log cos φ = 9.99454
log sec θ = 0.01053
a. c. log 800 = 7.09691

log v = 2.20708
v = 161.1 m. s.

$$\begin{aligned} \log v^2 &= 4.67494 \\ \log w &= 1.89625 \\ \text{a. c. } \log 2g &= 8.70730 \\ \text{a. c. } \log 1000 &= 7.00000 \\ \hline \log 189.89 &= 2.27849 \end{aligned}$$

The striking energy of the projectile is therefore 189.89 metre-tonnes.

$$\begin{aligned} \log v^2 &= 4.41416 \\ \log w &= 1.89625 \\ \text{a. c. } \log 2g &= 8.70730 \\ \text{a. c. } \log 1000 &= 7.00000 \\ \hline \log 104.16 &= 2.01771 \end{aligned}$$

The striking energy of the projectile is therefore 104.16 metre-tonnes.

If the breach is to be 6 metres high and 20 metres broad the volume of masonry to be demolished is 210 cubic metres.

According to the experiments made by the Prussians at Graudenz in 1875 there are required 132 metre-tonnes of energy for each cubic metre to be demolished. The total energy necessary to effect the breach is therefore

$$210 \times 132 = 27720 \text{ metre-tonnes.}$$

The number of projectiles that must hit the target is, then, as follows:

$$\text{Fired from position A, } \frac{27720}{189.99} = 146$$

$$\text{Fired from position B, } \frac{27720}{104.16} = 266$$

To determine the number of shots that must be fired from each position to effect the breach we must know the probability of hitting a rectangle 6 m by 20 m from each of the two positions. The mean vertical and horizontal quadratic deviations for the gun in question are known by previous experiments to be as follows for the two positions:

Position A.	*Position B.*
$E_x = 4.74$ m	$E_x = 2.18$ m
$E_y = 2.18$ m	$E_y = 0.83$ m

The horizontal, or lateral, deviations (E_y) are so small relatively to the breadth of the target that we may consider the probability of the vertical zone as unity. The probability for the horizontal zone is found from the table on page 225 of the Handbook. We have for

Position A.	*Position B.*
$\frac{6}{E_x} = 1.27$	$\frac{6}{E_x} = 2.75$
$P = .4745$	$P = .8308$
$\frac{146}{.4745} = 308$	$\frac{266}{.8308} = 321$

It will therefore require at least 308 shots from position A and 321 from position B to effect the breach. The position A is by these calculations slightly superior to B from a ballistic point of view. The above computations agree substantially with those made by La Llave.

PROBLEM FORTY-SIX.

To determine the elements of a trajectory for a given projectile which shall pass through two given points.

Solution: Let x, y and x_1, y_1 be the two given points, and ε, ε_1 their angular elevations (or depressions) above (or below) the level of the gun. Then

$$\tan \varepsilon = \frac{y}{x}$$

and

$$\tan \varepsilon_1 = \frac{y_1}{x_1}$$

From equation (53) we have

$$\tan \varepsilon = \frac{\tan \varphi}{A}(A - a)$$

and

$$\tan \varepsilon_1 = \frac{\tan \varphi}{A}(A - a_1)$$

In these equations a is to be taken from the "A" column with the argument

$$z = \frac{x}{C}$$

and a_1 with the argument

$$z_1 = \frac{x_1}{C}$$

Solving for A and $\tan \varphi$ we have

$$A = \frac{a_1 \tan \varepsilon - a \tan \varepsilon_1}{\tan \varepsilon - \tan \varepsilon_1}$$

and

$$\tan \varphi = \frac{A \tan \varepsilon}{A - a} = \frac{A \tan \varepsilon_1}{A - a_1}$$

Knowing C, φ and A the remaining elements of the trajectory can be computed by Problem Three. It will be observed that all the formulas of this problem are independent of the muzzle velocity.

Example 1. $C = 2$, $x = 1800$ ft., $y = 50$ ft., $x_1 = 2200$ ft., $y_1 = -10$ ft.

We find

$$z = 900 \qquad \tan \varepsilon = \frac{1}{36}$$
$$a = .04652$$
$$z_1 = 1100 \qquad \tan \varepsilon_1 = -\frac{1}{220}$$
$$a_1 = .05722$$

Substituting these numbers in the expressions for A and tan φ we have

$$A = \frac{220 \times .05722 + 36 \times .04652}{220 + 36} = .055715$$

$$\tan \varphi = \frac{.055715}{36 \times 009195} = .16831$$

$$\therefore \varphi = 9^\circ\ 31'$$

Next by the formula

$$V = 800 \left(\frac{A\ C}{\sin 2\ \varphi} \right)^{\frac{1}{2}}$$

we find

$$V = 466.6 \text{ f. s.}$$

All the other elements can now be computed by methods already explained and illustrated.

By the parabolic theory the expressions for tan φ and V are

$$\tan \varphi = \frac{x_1 \tan \varepsilon - x \tan \varepsilon_1}{x_1 - x}$$

and

$$V^2 = \frac{g\,(x_1 - x)}{2 \cos^2 \varphi\,(\tan \varepsilon - \tan \varepsilon_1)}$$

These formulas give with the data of this example

$$\varphi = 9^\circ\ 49'\ 40''$$
$$V = 452.7 \text{ f. s.}$$

Solution for metric units.

With metric units z and z_1 will be computed by Problem One.

u	*D*	*A*	*T′*	*D*	log *C′*	*D*	log *C″*	*D*	log *C‴*	*D*
					—10		—10			
796.3	−3.7	0.00504	0.125	126	5.7026	13	8.30352	34	1.5066	−6
792.6	3.7	.01011	0.251	127	5.7039	14	8.30386	34	1.5060	7
788.9	3.7	.01521	0.378	127	5.7053	13	8.30420	35	1.5053	7
785.2	3.6	.02035	0.505	128	5.7066	14	8.30455	34	1.5046	7
781.6	3.7	.02552	0.633	128	5.7080	14	8.30489	35	1.5039	6
777.9	3.6	.03072	0.761	129	5.7094	13	8.30524	35	1.5033	7
774.3	3.6	.03595	0.890	129	5.7107	14	8.30559	35	1.5026	7
770.7	3.6	.04122	1.019	130	5.7121	13	8.30594	35	1.5019	6
767.1	3.6	.04652	1.149	131	5.7134	14	8.30629	35	1.5013	7
763.5	3.6	.05186	1.280	131	5.7148	14	8.30664	35	1.5006	7
759.9	3.5	.05722	1.411	132	5.7162	14	8.30699	36	1.4999	6
756.4	3.5	.06262	1.543	133	5.7176	13	8.30735	35	1.4993	7
752.9	3.5	.06805	1.676	133	5.7189	14	8.30770	36	1.4986	7
749.4	3.5	.07352	1.809	134	5.7203	14	8.30806	36	1.4979	6
745.9	3.5	.07902	1.943	134	5.7217	14	8.30842	36	1.4972	6
742.4	3.5	.0845	2.077	135	5.7231	14	8.30878	36	1.4966	7
738.9	3.4	.0901	2.212	136	5.7245	13	8.30914	36	1.4959	7
735.5	3.5	.0957	2.348	136	5.7358	14	8.30950	36	1.4952	6
732.0	3.4	.1013	2.484	137	5.7272	14	8.30986	37	1.4946	7
728.6	3.4	.1070	2.621	138	5.7286	14	8.31023	37	1.4939	7
725.2	3.4	.1127	2.759	138	5.7300	14	8.31060	37	1.4932	6
721.8	3.4	.1185	2.897	139	5.7314	14	8.31097	37	1.4926	7
718.4	3.3	.1243	3.036	139	5.7328	14	8.31134	37	1.4919	6
715.1	3.3	.1301	3.175	140	5.7342	14	8.31171	37	1.4913	7
711.8	3.4	.1360	3.315	141	5.7356	15	8.31208	37	1.4906	7
708.4	3.3	.1419	3.456	142	5.7371	14	8.31245	38	1.4899	6
705.1	3.3	.1478	3.598	142	5.7385	14	8.31283	37	1.4893	7
701.8	3.2	.1538	3.740	143	5.7399	14	8.31320	38	1.4886	6
698.6	3 3	.1598	3.883	143	5.7413	14	8.31358	38	1.4880	7
695.3	3.3	.1659	4.026	144	5.7427	14	8.31396	38	1.4873	7
692.0	3.2	.1719	4.170	145	5.7441	15	8.31434	38	1.4866	6
688.8	3.2	.1781	4.315	145	5.7456	14	8.31472	38	1.4860	7
685.6	3.2	.1842	4.460	147	5.7470	14	8.31510	39	1.4853	6
682.4	3.2	.1905	4.607	146	5.7484	14	8.31549	38	1.4847	7
679.2	3.2	.1967	4.753	148	5.7498	15	8.31587	39	1.4840	7
676.0	3.1	.2030	4.901	148	5.7513	14	8.31626	39	1.4833	6
672.9	3.2	.2093	5.049	149	5.7527	14	8.31665	39	1.4827	7
669.7	3.1	.2157	5.198	150	5.7541	15	8.31704	39	1.4820	6
666.6	3.1	.2221	5.348	150	5.7556	14	8.31743	39	1.4814	7
663.5	3.1	.2286	5.498	151	5.7570	14	8.31782	39	1.4807	7
660.4	3.1	.2350	5.649	152	5.7584	15	8.31821	40	1.4800	6
657.3	3.0	.2416	5.801	152	5.7599	14	8.31861	40	1.4794	7
654.3	3.1	.2482	5.953	153	5.7613	15	8.31901	40	1.4787	6
651.2	3.0	.2548	6.106	154	5.7628	14	8.31941	40	1.4781	7
648.2	3.0	.2614	6.260	155	5.7642	15	8.31981	40	1.4774	6

u	*D*	*A*	*T′*	*D*	log *C′*	*D*	log *C″*	*D*	log *C‴*	*D*
					—10		—10			
645.2	−3.0	0.26820	6.415	156	5.7657	14	8.32021	40	1.4768	−7
642.2	3.0	.27495	6.571	156	5.7671	15	8.32061	41	1.4761	6
639.2	3.0	.28174	6.727	156	5.7686	14	8.32102	40	1.4755	7
636.2	3.0	.28858	6.883	158	5.7700	15	8.32142	41	1.4748	6
633.2	2.9	.29546	7.041	158	5.7715	15	8.32183	41	1.4742	7
630.3	3.0	.30238	7.199	159	5.7730	15	8.32224	41	1.4735	6
627.3	2.9	.30935	7.358	160	5.7745	14	8.32265	41	1.4729	7
624.4	2.9	.31637	7.518	161	5.7759	15	8.32306	41	1.4722	6
621.5	2.9	.32343	7.679	161	5.7774	15	8.32347	41	1.4716	7
618.6	2.9	.33054	7.840	162	5.7789	15	8.32388	42	1.4709	6
615.7	2.9	.33770	8.002	163	5.7804	15	8.32430	41	1.4703	7
612.8	2.8	.34490	8.165	164	5.7819	14	8.32471	42	1.4696	6
610.0	2.9	.35215	8.329	164	5.7833	15	8.32513	42	1.4690	7
607.1	2.8	.35944	8.493	165	5.7848	15	8.32555	42	1.4683	6
604.3	2.8	.36678	8.658	166	5.7863	15	8.32597	42	1.4677	6
601.5	2.8	.37417	8.824	166	5.7878	15	8.32639	42	1.4671	7
598.7	2.8	.38161	8.990	168	5.7893	15	8.32681	43	1.4664	6
595.9	2.8	.38910	9.158	168	5.7908	15	8.32724	42	1.4658	6
593.1	2.8	.39664	9.326	169	5.7923	14	8.32766	43	1.4652	7
590.3	2.7	.40422	9.495	170	5.7937	15	8.32809	43	1.4645	6
587.6	2.7	.41186	9.665	170	5.7952	15	8.32852	43	1.4639	6
584.9	2.8	.41955	9.835	172	5.7967	15	8.32895	44	1.4633	6
582.1	2.7	.42728	0.007	172	5.7982	15	8.32939	43	1.4627	7
579.4	2.7	.43507	0.179	173	5.7997	15	8.32982	44	1.4620	6
576.7	2.7	.44291	0.352	174	5.8012	15	8.33026	44	1.4614	6
5'4.0	2.7	.45080	0.526	174	5.8027	15	8.33070	44	1.4608	7
571.3	2.7	.45875	0.700	176	5.8042	16	8.33114	44	1.4601	6
568.6	2.6	.46674	0.876	176	5.8058	15	8.33158	45	1.4595	6
566.0	2.7	.47479	1.052	177	5.8073	15	8.33203	44	1.4589	7
563.3	2.6	.48289	1.229	178	5.8088	15	8.33247	45	1.4582	6
560.7	2.6	.49105	1.407	179	5.8103	15	8.33292	45	1.4576	6
558.1	2.6	.49926	1.586	179	5.8118	16	8.33337	45	1.4570	6
555.5	2.6	.50752	1.765	181	5.8134	15	8.33382	45	1.4564	7
552.9	2.6	.51584	1.946	181	5.8149	15	8.33427	45	1.4557	6
550.3	2.6	.52421	2.127	182	5.8164	15	8.33472	45	1.4551	6
547.7	2.5	.53264	2.309	183	5.8179	16	8.33517	45	1.4545	7
545.2	2.6	.54112	2.492	184	5.8195	15	8.33562	46	1.4538	6
542.6	2.5	.54966	2.676	184	5.8210	16	8.33608	46	1.4532	6
540.1	2.5	.55826	2.860	186	5.8226	15	8.33654	45	1.4526	7
537.6	2.5	.56691	3.046	186	5.8241	15	8.33699	46	1.4519	6
535.1	2.5	.57562	3.232	188	5.8256	16	8.33745	46	1.4513	6
532.6	2.5	.58439	3.420	188	5.8272	15	8.33791	47	1.4507	6
530.1	2.4	.59322	3.608	189	5.8287	16	8.33838	46	1.4501	6
527.7	2.5	.60211	3.797	190	5.8303	15	8.33884	47	1.4494	7
525.2	2.5	.61105	3.987	191	5.8318	16	8.33931	47	1.4488	6

z	*u*	*D*	*A*	*D*	g *C′*	*D*	log *C″*	*D*	log *C‴*	*D*
					10—		10—			
9100	522.7	−2.4	0.62005	907	8334	15	8.33978	46	1.4482	−6
9200	520.3	2.4	.62912	913	8349	16	8.34024	47	1.4476	6
9300	517.9	2.4	.63825	918	8365	16	8.34071	48	1.4470	6
9400	515.5	2.4	.64743	925	8381	15	8.34119	47	1.4464	6
9500	513.1	2.4	.65668	931	8396	16	8.34166	48	1.4458	7
9600	510.7	2.4	.66599	937	8412	16	8.34214	47	1.4451	6
9700	508.3	2.4	.67536	944	8428	16	8.34261	48	1.4445	6
9800	505.9	2.4	.68480	950	8444	15	8.34309	49	1.4439	6
9900	503.5	2.3	.69430	956	8459	16	8.34358	49	1.4433	6
10000	501.2	2.3	.70386	962	8475	16	8.34406	49	1.4427	6
10100	498.9	2.4	.71348	969	8491	16	8.34455	49	1.4421	6
10200	496.5	2.3	.72317	976	8507	16	8.34504	49	1.4415	6
10300	494.2	2.3	.73293	982	8523	16	8.34553	49	1.4409	6
10400	491.9	2.3	.74275	989	8539	16	8.34602	49	1.4403	7
10500	489.6	2.3	.75264	995	8555	15	8.34651	50	1.4396	6
10600	487.3	2.2	.76259	1002	8570	16	8.34701	49	1.4390	6
10700	485.1	2.3	.77261	1009	8586	16	8.34750	50	1.4384	6
10800	482.8	2.3	.78270	1015	8602	16	8.34800	50	1.4378	6
10900	480.5	2.2	.79285	1023	8618	16	8.34850	50	1.4372	6
11000	478.3	2.2	.80308	1030	8634	16	8.34900	50	1.4366	6
11100	476.1	2.3	.81338	1036	8650	16	8.34950	50	1.4360	6
11200	473.8	2.2	.82374	1044	8666	16	8.35000	50	1.4354	6
11300	471.6	2.2	.83418	1051	8682	16	8.35050	51	1.4348	6
11400	469.4	2.2	.84469	1058	8698	16	8.35101	51	1.4342	6
11500	467.2	2.2	.85527	1065	8714	17	8.35152	50	1.4336	6
11600	465.0	2.2	.86592	1072	8731	16	8.35202	51	1.4330	6
11700	462.8	2.1	.87664	1079	8747	16	8.35253	51	1.4324	6
11800	460.7	2.2	.88743	1086	8763	16	8.35304	52	1.4318	6
11900	458.5	2.1	.89829	1094	8779	16	8.35356	51	1.4312	6
12000	456.4	2.1	.90923	1102	8795	16	8.35407	51	1.4306	6
12100	454.3	2.1	.92025	1109	8811	17	8.35458	52	1.4300	6
12200	452.2	2.1	.93134	1116	8828	16	8.35510	52	1.4294	6
12300	450.1	2.1	.94250	1124	8844	16	8.35562	52	1.4288	6
12400	448.0	2.1	.95374	1132	8860	16	8.35614	52	1.4282	6
12500	445.9	2.1	.96506	1139	8876	17	8.35666	52	1.4276	5
12600	443.8	2.1	.97645	1147	8893	16	8.35718	53	1.4271	6
12700	441.7	2.0	.98792	1155	8909	16	8.35771	52	1.4265	6
12800	439.7	2.1	.99947	1162	8925	17	8.35823	53	1.4259	6
12900	437.6	2.0	1.01109	1171	8942	16	8.35876	53	1.4253	6
13000	435.6	2.0	1.02280	1179	8958	17	8.35929	53	1.4247	6
13100	433.6	2.1	1.03459	1187	8975	16	8.35982	53	1.4241	6
13200	431.5	2.1	1.04646	1195	8991	17	8.36035	54	1.4235	6
13300	429.4	2.0	1.05841	1203	9008	16	8.36089	53	1.4229	6
13400	427.4	2.0	1.07044	1211	9024	17	8.36142	54	1.4223	6
13500	425.4	1.9	1.08255	1220	9041	17	8.36196	54	1.4217	5
13600	423.5	2.0	1.09475	1238	9058	16	8.36250	54	1.4212	6
13700	421.5	2.0	1.10703	1236	9074	17	8.36304	54	1.4206	6
13800	419.5	2.0	1.11939	1245	9091	16	8.36358	54	1.4200	6
13900	417.5	1.9	1.13184	1254	9107	17	8.36412	55	1.4194	6
14000	415.6	1.9	1.14438	1263	0124	17	8.36467	55	1.4188	5

TABLE 3.—*Values of* $\frac{\delta_t}{\delta}$ *for temperature and pressure of atmosphere two-thirds saturated with moisture.*

F	28 in.	29 in.	30 in.	31 in.	*F*	28 in.	29 in.	30 in.	31 in.
0°	0.945	0.912	0.882	0.853	52°	1.054	1.018	0.984	0.953
1	0.947	0.914	0.884	0.855	53	1.056	1.020	0.986	0.954
2	0.949	0.916	0.886	0.857	54	1.058	1.022	0.988	0.956
3	0.951	0.918	0.888	0.859					
4	0.953	0.920	0.890	0.861	55	1.061	1.024	0.990	0.958
5	0.955	0.922	0.892	0.863	56	1.063	1.026	0.992	0.960
6	0.957	0.924	0.893	0.865	57	1.065	1.028	0.994	0.962
7	0.959	0.926	0.895	0.867	58	1.067	1.030	0.996	0.964
8	0.962	0.928	0.897	0.869	59	1.069	1.032	0.998	0.966
9	0.964	0.930	0.899	0.870	60	1.071	1.034	1.000	0.968
10	0.966	0.932	0.901	0.872	61	1.073	1.037	1.002	0.970
11	0.968	0.935	0.903	0.874	62	1.075	1.039	1.004	0.972
12	0.970	0.937	0.905	0.876	63	1.078	1.041	1.006	0.974
13	0.972	0.939	0.907	0.878	64	1.080	1.043	1.008	0.975
14	0.974	0.941	0.909	0.880	65	1.082	1.045	1.010	0.977
15	0.976	0.943	0.911	0.882	66	1.084	1.047	1.012	0.979
16	0.978	0.945	0.913	0.884	67	1.086	1.049	1.014	0.981
17	0.981	0.947	0.915	0.886	68	1.088	1.051	1.016	0.983
18	0.983	0.949	0.917	0.888	69	1.090	1.053	1.018	0.985
19	0.985	0.951	0.919	0.890	70	1.092	1.055	1.020	0.987
20	0.987	0.953	0.921	0.891	71	1.094	1.057	1.022	0.989
21	0.989	0.955	0.923	0.893	72	1.097	1.059	1.024	0.991
22	0.991	0.957	0.925	0.895	73	1.099	1.061	1.025	0.993
23	0.993	0.959	0.927	0.897	74	1.101	1.063	1.027	0.995
24	0.995	0.961	0.929	0.899	75	1.103	1.066	1.029	0.996
25	0.997	0.963	0.931	0.901	76	1.105	1.067	1.031	0.998
26	1.000	0.965	0.933	0.903	77	1.107	1.069	1.033	1.000
27	1.002	0.967	0.935	0.905	78	1.109	1.071	1.035	1.002
28	1.004	0.969	0.937	0.907	79	1.111	1.073	1.037	1.004
29	1.006	0.971	0.939	0.909	80	1.113	1.075	1.039	1.006
30	1.008	0.973	0.941	0.911	81	1.116	1.077	1.041	1.008
31	1.010	0.975	0.943	0.912	82	1.118	1.079	1.043	1.010
32	1.012	0.977	0.945	0.914	83	1.120	1.081	1.045	1.012
33	1.014	0.979	0.947	0.916	84	1.122	1.083	1.047	1.014
34	1.016	0.981	0.949	0.918	85	1.124	1.085	1.049	1.016
35	1.018	0.983	0.951	0.920	86	1.126	1.088	1.051	1.017
36	1.021	0.986	0.953	0.922	87	1.128	1.090	1.053	1.019
37	1.023	0.988	0.955	0.924	88	1.130	1.092	1.055	1.021
38	1.025	0.990	0.957	0.926	89	1.132	1.094	1.057	1.023
39	1.027	0.992	0.958	0.928	90	1.135	1.096	1.059	1.025
40	1.029	0.994	0.960	0.930	91	1.137	1.098	1.061	1.027
41	1.031	0.996	0.962	0.932	92	1.139	1.100	1.063	1.029
42	1.033	0.998	0.964	0.933	93	1.141	1.102	1.065	1.031
43	1.035	1.000	0.966	0.935	94	1.143	1.104	1.067	1.033
44	1.037	1.002	0.968	0.937	95	1.145	1.106	1.069	1.035
45	1.040	1.004	0.970	0.939	96	1.147	1.108	1.071	1.037
46	1.042	1.006	0.972	0.941	97	1.149	1.110	1.073	1.038
47	1.044	1.008	0.974	0.943	98	1.151	1.112	1.075	1.040
48	1.046	1.010	0.976	0.945	99	1.155	1.114	1.077	1.042
49	1.048	1.012	0.978	0.947	100	1.157	1.116	1.079	1.044
50	1.050	1.014	0.980	0.949					
51	1.052	1.016	0.982	0.951					

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